NONLINEAR SURF ZONE WAVE PROPERTIES AS ESTIMATED FROM BOUSSINESQ MODELLING : RANDOM WAVES AND COMPLEX BATHYMETRIES

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The present work aims at investigating the ability of Boussinesq-type equations and breaking-wave parameterizations to reproduce nonlinear properties of surf zone waves. We compare results produced by two different breaking models : those proposed by Kennedy et al. (2000) and by Cienfuegos et al. (2005). Both breaking strategies are implemented in a fully nonlinear and weakly dispersive Boussinesq code (Cienfuegos et al., 2006a,b). In the first part we calibrate model parameters on the spilling regular wave experiment conducted by Ting and Kirby (1994). In the 2nd part, we apply the breaking Boussinesq models on a new laboratory experiment on random waves propagating over uneven bathymetries.

1. Introduction

Over the last 15 years important practical improvements for Boussinesq-type models have been introduced extending their applicability range into deeper waters and into the surf and swash zones. The breaking processes have been modelled by means of an *ad-hoc* extra term, usually written with a diffusive mathematical form, added in the momentum conservation equation of the inviscid (non dissipative) set of Boussinesq equations. Essentially, three different approaches to parameterize breaking terms can be found in the literature : i) roller-based models (e.g. Brocchini et al., 1992; Schäffer et al., 1993), the turbulent eddy viscosity analogy (e.g. Zelt, 1991; Kennedy et al., 2000), and iii) models accounting for the vorticity generation under breakers (e.g. Veeramony and Svendsen, 2000; Musumeci et al.,

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2005). An alternative approach was recently proposed by Cienfuegos et al. (2005) where a diffusive-type term is also introduced in the mass conservation equation. These authors use experimental and theoretical results on shallow water breakers to obtain a physical interpretation for mass and momentum mixing coefficients and show good agreement between computed and measured free surface profiles in a regular wave surf zone.

Eventough the available wave breaking formulations have been extensively calibrated and validated on regular wave experiments and uniform beach slopes, in nature, propagation and breaking is a random process. Consequently, there is still a need for testing these models on realistic situations where random waves propagate over uneven bathymetries. Ozanne et al. (2000) and Bayram and Larson (2000) reported some of the few examples where computed free surface profiles were compared against field measurements. Nevertheless, their numerical comparisons were somehow mistrusted by the poor nonlinear characteristics that were embedded in the chosen low order version of Boussinesq-type equations. It is worth noting that those authors didn't show results on the nonlinear energy transfer produced by their models.

The present work aims at investigating the ability of Boussinesq equations and breaking-wave parameterizations to reproduce nonlinear properties of surf zone waves and nonlinear energy transfer in the shoaling and surf zones. We focus in particular on wave height, skewness and asymmetry since it appears that these properties are paramount for sediment transport prediction (see for instance Drake and Calantoni, 2001). We investigate the application of two breaking models, those proposed by Kennedy et al. (2000) (KENN hereafter) and the one developed by Cienfuegos et al. (2005) (CIEN in the following). Both parameterizations are implemented in the fully nonlinear finite volume code SERR-1D (see Cienfuegos et al., 2006a,b, for details).

In the first part of the paper we calibrate breaking models and compare their results for the regular spilling breaking experiment reported by Ting and Kirby (1994). In the second part, we move on to an experimental random wave test keeping the same parameter values. We investigate model predictions in terms of free surface profiles in time domain and the nonlinear energy evolution and transfer in the frequency domain.

2. Model Calibration on a Regular Wave Experiment

2.1. *Breaking-wave models*

Depth-averaged mass and momentum conservation equations can be written in the following generic form,

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) - D_h = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} + g\frac{\partial h}{\partial x} + \Gamma_d - \frac{1}{h}D_{hu} = 0, \tag{2}$$

where h is the water depth, u is the depth-averaged horizontal velocity, D_h and D_{hu} represent extra terms accounting for energy dissipation by breaking, Γ_d groups all high-order dispersive (non-hydrostatic) terms, and g is the gravitational acceleration; variables x and t denote space and time coordinates. In surf zone Boussinesq models, a breaking criterion must be adopted to turn on/off breaking dissipation, and wave crests need to be followed since a wave-by-wave approach must be considered (see for instance Kirby, 2003).

KENN model introduces breaking effects in the momentum equation only, i.e. $D_h = 0$. The remaining breaking term reads,

$$D_{hu} = \frac{\partial}{\partial x}\left(\nu_{hu}\frac{\partial hu}{\partial x}\right), \tag{3}$$

with the eddy viscosity coefficient, ν_{hu}, given as,

$$\nu_{hu} = B\,\delta_b^2\,h\,\frac{\partial h}{\partial t}, \tag{4}$$

where δ_b is a constant mixing length coefficient, and B is the parameter which activates breaking terms avoiding an impulsive behaviour. The breaking initiation/cessation criterion is based on threshold values for the time derivative of the free surface (see Kennedy et al., 2000, for details).

On the other hand, CIEN breaking model includes a mass conservative diffusivity term in the continuity equation intended to mimic local mass redistribution inside the roller and mixing layer effects. While the breaking induced momentum term is written as in (3), the additional mixing term in eq. (1) reads,

$$D_h = \frac{\partial}{\partial x}\left(\nu_h\frac{\partial h}{\partial x}\right). \tag{5}$$

In this model, mass and momentum diffusivity coefficients, ν_h and ν_{hu}, are written as,

$$\nu_h = -\delta_h \frac{c\,d}{\tan\Phi}\ \Lambda(X/l_r), \quad \text{for } 0 \leq X \leq l_r, \tag{6}$$

$$\nu_{hu} = -\delta_{hu} \frac{c\,d}{\tan\Phi}\ \Lambda(X/l_r), \quad \text{for } \quad 0 \leq X \leq l_r, \tag{7}$$

where δ_h and δ_{hu} are order-one coefficients, c is the wave celerity, d is the mean water depth and Φ is the breaker angle. Λ is a self-similar shape function that locally distributes diffusivity over the breaker front ensuring that extra terms are conservative over a breaking event. This form relies on experimental results produced by Svendsen et al. (2000) and is written as,

$$\Lambda(X/l_r) = \exp\left(\frac{X}{l_r} - 1\right)\left[\left(\frac{X}{l_r} - 1\right) + \left(\frac{X}{l_r} - 1\right)^2\right],$$

with X a local coordinate system moving with the wave ($X = 0$ is located at the crest), and l_r the roller horizontal length. The different parameters have been scaled using empirical and theoretical knowledge on quasi-steady shallow water breakers and do have a physical interpretation (see Cienfuegos et al., 2005, for details).

In the present work we simplify the model taking δ_{hu} and Φ as constants and assuming that $\delta_h = 0.1\delta_{hu}$. Finally, the roller length is related to local wave properties using Cointe and Tulin (1994) theory of steady breakers calibrated on hydraulic jumps in similarity with surf zone waves (see Cienfuegos et al., 2004). It follows that $l_r/d = \frac{0.97}{\tan\Phi}(1-\gamma)$ introducing the breaker index $\gamma = H/d$, with H being the local wave height.

2.2. *Optimal parameter values*

The numerical values of model parameters are calibrated on the regular wave spilling breaking data reported by Ting and Kirby (1994). Cnoidal waves with incident height $H_0 = 0.127$ m and period $T = 2.0$ seconds propagate towards a uniform beach of 1:35 slope. The still water level is fixed at $h_0 = 0.4$ m in the horizontal part of the flume (see Fig. 1). Phase-averaged time series of free surface elevation are vailable at 21 locations, before and after breaking.

To test and calibrate the breaking models we define an error index based on measured and computed time-domain wave properties over the

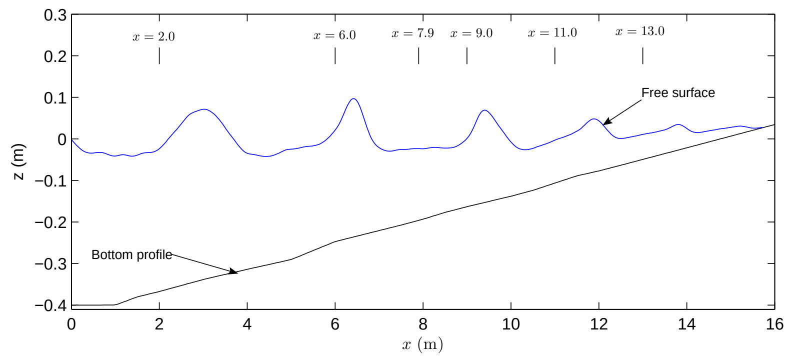

Figure 1. Definition sketch for Ting and Kirby (1994) spilling breaking experiment.

whole domain. Following Kennedy et al. (2000) we introduce left-right asymmetry as,

$$As = \frac{\langle \mathcal{H}^3(\eta - \overline{\eta}) \rangle}{\langle (\eta - \overline{\eta})^2 \rangle^{3/2}}, \tag{8}$$

where $\langle \cdot \rangle$ is the mean operator, $\mathcal{H}$ is the Hilbert transform, η is the free surface elevation relative to the still water level and overbar denotes time averaging. Similarly, crest-trough asymmetry or wave skewness is estimated as,

$$Sk = \frac{\langle (\eta - \overline{\eta})^3 \rangle}{\langle (\eta - \overline{\eta})^2 \rangle^{3/2}}. \tag{9}$$

A root mean square error (RMSE) can be computed for wave height, asymmetry and skewness; the calibration process consists in minimizing the average error for these three properties.

In the left panel of Fig. 2 we show optimal results obtained using CIEN model. The best agreement between computed and measured properties is produced for $\Phi = 17.0°$, $\gamma = 0.7$ and $\delta_{hu} = 5.0$ with an average RMSE of 14.2 % (RMSE for H, As and Sk being respectively 7.3 %, 24.1 % and 11.3 %). The biggest error occurs for wave asymmetry in the vicinity of the breaking point in the transition zone. Nevertheless, the model succeeds in accurately reproducing nonlinear wave evolution in the inner surf zone. It is worth noting that breaking terms are activated when the local breaker angle reaches 30°, conversely, breaking stops if it falls below an angle of 8°.

Results obtained using KENN breaking model are shown in the right panel of Fig. 2. Even though we closely follow the methodology detailed in

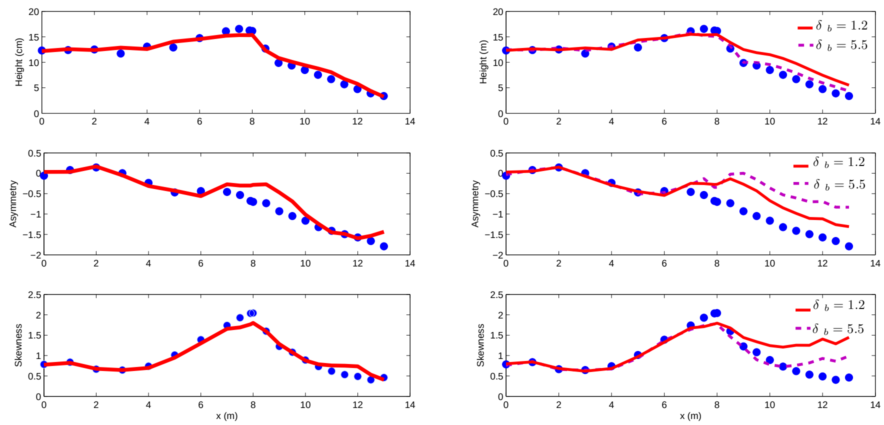

Figure 2. Comparison between measured (•) and computed (−, −−) wave properties for Ting and Kirby (1994) experiment. Left panel: CIEN breaking model. Right panel: KENN breaking model.

Kennedy et al. (2000), the numerical values for parameter δ_b in eq. (4) and for the iniciation criterion of breaking are to be adjusted in the calibration. Indeed, in the present example the critical value for the free surface time derivative had to be lowered to $\eta_t^I = 0.55\sqrt{g\,d_0}$ in order to match the breaking point (d_0 is the local still water depth).

We conducted several computations using different values for δ_b because this coefficient sets the intensity of breaking energy dissipation. In Fig. 2 we show results for $\delta_b = 1.2$, which is the value recommended by Kennedy et al. (2000), and for $\delta_b = 5.5$.

The default value, $\delta_b = 1.2$, produces an over estimation of wave height in the inner surf zone in agreement with results published in Kennedy et al. (2000). On the other hand, even though the slope of the wave asymmetry curve is in agreement with the measured one, there is a vertical shift in the absolute value of this property all over the surf zone. Finally, it is seen that wave skewness in the inner surf zone is heavily overestimated by KENN model. The average RMSE for H, As and Sk is nearly 31.6 % (The RMSE for each one is respectively 16.6 %, 39.9 % and 38.3 %).

In order to improve wave height estimates in the inner surf zone, we set $\delta_b = 5.5$ to force higher energy dissipation. This parameter value allows to reduce the RMSE in wave height to 7.6 %, which is similar to the error we got using CIEN model. The RMSE in Sk is also smaller than before being now of nearly 19.2 %. Unfortunately, the RMSE in wave asymmetry, As, is considerably larger than before (61.1 %) which means that by forcing the model with such a high δ_b, the characteristic surf zone saw-tooth wave profile is completely lost. The average RMSE error for the three properties is now of nearly 29.3 %, i.e. not so different from the one we obtained by applying the recommended value $\delta_b = 1.2$. Hence, it seems that the default value for this parameter represents a reasonable compromise. Indeed, improving the estimate of wave height in the inner surf zone by forcing δ_b will inevitably increase the error in wave asymmetry predictions.

In what follows we will use $\delta_b = 1.2$ as suggested by Kennedy et al. (2000) and we freeze the calibrated parameter values for CIEN model. In the following we move on to a more realistic situation where random waves propagate over uneven bathymetries.

3. Application to Random Waves

3.1. *Experimental set-up*

Our experiments were carried out in a flume 36 m long and 55 cm wide equipped with a piston wave generator. The water depth at rest is 55.3 cm. The mean overall slope is approximately 1/40 (see Fig. 3). The sloping bottom consists of a loose material for which the low density ($1.19\,\mathrm{g\ cm^{-3}}$) and the median diameter $d_{50} = 0.6$ mm allow representative sheet-flow and suspension regimes. The experiments were designed in order that the Froude number is of the same magnitude as those of natural environments under mild wave forcing. Length scales are roughly 1/10 and thus time scales roughly 1/3. The Shields number in the shoaling zone and the Rouse number in the breaking zone (ratio of turbulent agitation to the settling velocity

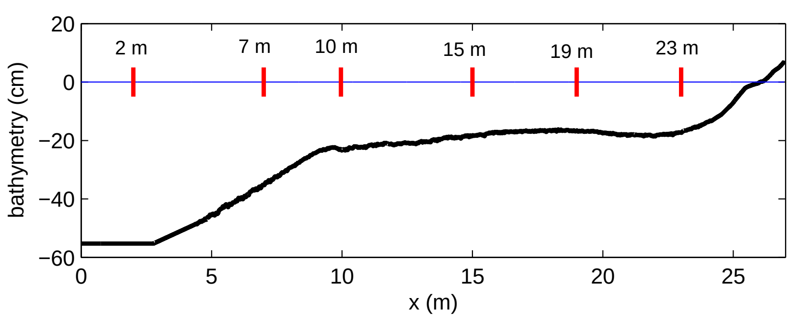

Figure 3. Bottom profile and wave gauge locations for the random wave experiment.

of the sediment) are also of the same magnitude as those of natural environments. Irregular waves are generated according to a JONSWAP spectrum (peak enhancement factor =3.3 and peak frequency at $f_p = 0.33$ Hz). We checked that these waves conform to the expected spectrum and that they follow a Rayleigh distribution at 2 m downstream of the wave maker. Starting from a strongly disrupted initial condition, the beach profile was formed by the breaking waves in about 15 hours. During the following 150 hours of the experiment, the beach profile did not change in the mean. This confirms that equilibrium state between the bed morphology and its hydrodynamical forcing can be reached in laboratory conditions. The form of the equilibrium profile is at the transition between a barred and a terraced beach, similar to the one obtained in a wave basin for spilling waves by Wang et al. (2003). The large waves tend to break where the bottom also breaks ($x = 10$m) from steep to mild slope. These large waves were observed to break up to the beach. Small waves tend to break very close to the shoreline. The large terrace creates a very challenging situation for numerical models. Waves that are at the threshold of breaking stop and start breaking all along this terrace.

3.2. *Model results*

The numerical model is now applied to the random wave experiment described above for the quasi-equilibrium beach configuration. The location of wave gauges where measurements are available is depicted in Fig. 3. The time serie of free surface elevation at the wave gauge located at $x = 2.0$ m is used to prescribe in the numerical model the incident random wave field at the seaward boundary. A moving shoreline condition, adapted from Lynett et al. (2002), is implemented on the right boundary (see Cienfuegos et al., 2006b, for details).

We perform computations over 1200 s using CIEN and KENN breaking models with fixed parameter values as calibrated in Subsection 2.2. Comparisons between computed and measured free surface elevations at different locations are presented in Fig. 4. It can be seen that wave deformation in the shoaling zone ($x = 2 - 10$ m) is accurately represented by the model thus confirming the good nonlinear properties of the chosen set of Boussinesq equations. Numerical predictions for waves propagating in the terrace-like part of the beach ($x = 10 - 19$ m) where the biggest waves continue breaking are also in good agreement with the experimental data. In the upper part of the beach, after the bar, most of the waves break and only minor differences (essentially phase-lag errors) can be observed between CIEN and KENN breaking models in this time window. In addition, H_{rms}, set-up and intra-phase statistics (aymmetry and skewness)

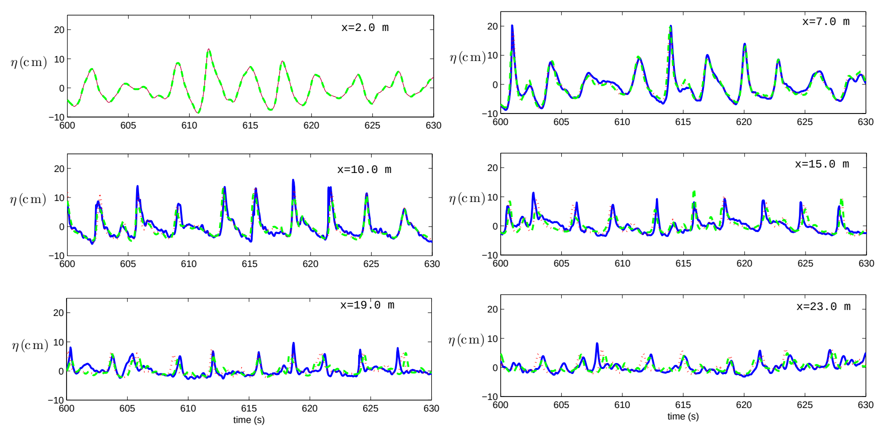

Figure 4. Time windows for measured and computed free surface at different spatial locations. (−) measured, (−−) computed with CIEN model, and (−.−) computed with KENN model.

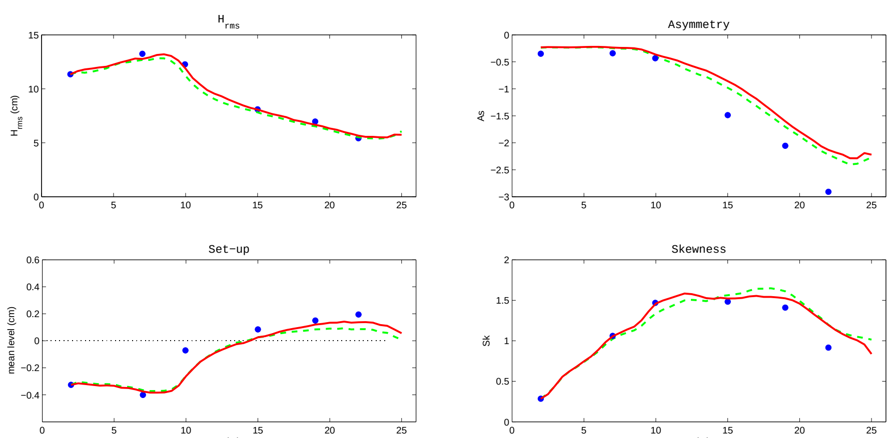

Figure 5. Wave properties. (•) measured, (−−) computed with CIEN model, and (−) computed with KENN model.

are plotted in Fig. 5. The overall agreement between computed and measured free surface statistics is considered adequate for this challenging test. In particular, H_{rms} and set-up evolution in the shoaling and surf zone is very well reproduced by both models. However, the computed set-up starts shorewards probably because the breaking iniciation criterion cannot accurately reproduce all types of breaking mechanisms. It is worth noting that CIEN model produces a slight underprediction of the set-up level in the inner zone. On the other hand, the numerical estimates of asymmetry and skewness produced by both breaking models are very similar unlike the situation we had in the regular wave experiment. There is a slight tendency to underestimate horizontal asymmetry in the inner surf zone while skewness prediction is quite reasonable. The randomness of the incident wave field and the averaging process used to compute wave statistics probably contribute to smear out model results in this case. In addition, it seems that wave reformation over the terrace-like bathymetry is over predicted probably because breaking terms are turned off too early. This could explain why wave asymmetry is not accurately reproduced inside the surf zone neither in CIEN nor in KENN models.

Finally, we perform a spectral analysis in order to study the nonlinear energy transfer inside the domain and results are presented in Fig. 6. As expected, for the first wave gauge located at $x = 2$ m the prescribed narrow

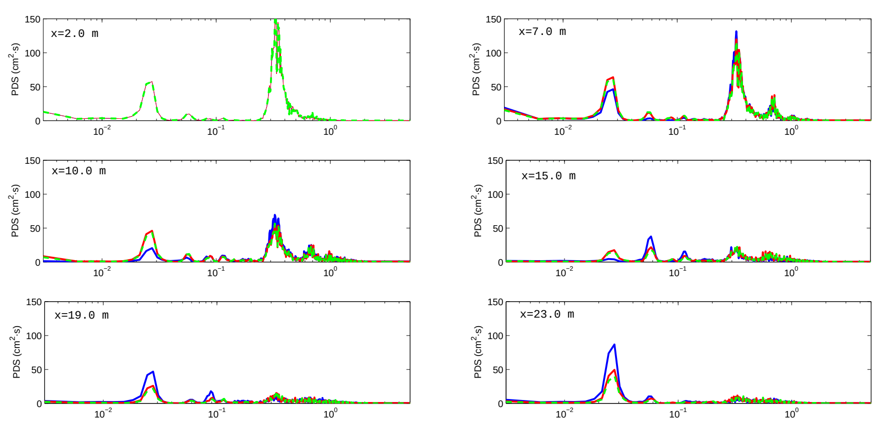

Figure 6. Spectral analysis : power density spectra. (−) measured, (−−) computed with CIEN model, and (−.−) computed with KENN model.

band JONSWAP spectrum is recovered centered at the peak frequency $f_p = 0.33$ Hz. However, a low frequecy peak at $f_p = 0.027$ Hz is also evident. Further analysis suggest that this low frequency motion corresponds to a nearly standing wave for which the wave-length is nearly equal to the flume's length. This is also sustained by the spatial evolution of the energy content at this frequency since maximum values are reached near the two edges of the flume while at $x = 15$ m the energy is much smaller. Antinodes of the standing wave are located near the channel edges, while the node is near its center. The numerical model underpredicts the energy content at the antinodes but overpredicts its value at the node. However it succeeds in reproducing the nonlinear harmonic generation for both, the infragravity wave and the incident narrow band Jonswap spectrum. Energy dissipation by breaking is also well reproduced and differences between predictions produced by CIEN and KENN models are mild.

4. Conclusions

In this work we implemented the wave-breaking parameterizations proposed by Kennedy et al. (2000) and Cienfuegos et al. (2005) on a fully nonlinear and weakly dispersive Boussinesq-type model. We calibrated model parameters on the regular wave experiment conducted by Ting and Kirby (1994) trying to minimize the joint RMSE on wave height, horizontal asymmetry and skewness. For this particular experiment, Cienfuegos et al. (2005) breaking model was able to provide an accurate prediction of these three quantities (joint RMSE of 14%). On the other hand, the error produced by Kennedy et al. (2000) model could not be reduced below 29 % mainly because decreasing the RMSE in wave heights in the inner surf zone produced inaccurate estimates of horizontal asymmetries.

In the second part, surf zone Boussinesq models were applied to a random wave experiment carried out in the LEGI's wave flume (36 m). A narrow band Jonswap spectrum propagated towards a terrace-like beach bathymetry. Comparisons between measured and predicted results showed an overall good agreement in the shoaling and surf zones. Unlike the regular wave case, the application of the two breaking models only produced minor differences. This could be attributed to the randomness of wave propagation since at one particular location inside the surf zone, wave statistics are computed over breaking and non breaking waves. On the other hand, over the quasi-horizontal terrace-like part of the beach, breaking seems to be prematurely turned off in both models (both model uses a similar breaking criterion). Consequently, wave reformation is over predicted by the numerical model and wave statistics is computed on a larger number of non breaking waves. This situation could also explain why different breaking models are producing rather similar results in this experiment.

Finally, a spectral analysis showed that breaking Boussinesq models were able to correctly reproduce nonlinear energy transfer in the shoaling and surf zones. Harmonic generation in the shoal and energy dissipation by breaking was accurately represented using both breaking parameterizations. On the other hand, a low-frequency nearly standing wave was generated in the wave flume. This infragravity motion was also captured by the numerical models showing only slight amplitude differences between measurements and computations. The question of infragravity wave generation under random wave fields needs to be investigated in more detail since long term beach morphology seems to respond to these low frequency forcing. For instance, in LEGI's random wave experiment beach bathymetry is in quasi-equilibrium and the observed infragravity wave is a nearly standing one.

Acknowledgments

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