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New Perspectives for Nonlinear Depth-Inversion of the Nearshore Using Boussinesq Theory

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Key Points:

- A new depth-inversion approach for the nearshore is proposed, based on a Boussinesq theory for quantifying nonlinear dispersion effects
- Unprecedented levels of accuracy (typically within 10%) are obtained in the surf zone over both planar and barred beaches
- Improvement over the linear wave theory method, which overestimates depths by 40% or more in surf zones (up to 80% at the shoreline)

Supporting Information:

Supporting Information may be found in the online version of this article.

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Abstract Accurately mapping the evolving bathymetry under energetic wave breaking is challenging, yet critical for improving our understanding of sandy beach morphodynamics. Though remote sensing is one of the most promising opportunities for reaching this goal, existing depth-inversion algorithms using linear approaches face major theoretical and/or technical issues in the surf zone, limiting their accuracy over this region. Here, we present a new depth-inversion approach relying on Boussinesq theory for quantifying nonlinear dispersion effects in nearshore waves. Using high-resolution datasets collected in the laboratory under diverse wave conditions and beach morphologies, we demonstrate that this approach results in enhanced levels of accuracy in the surf zone (errors typically within 10%) and presents a major improvement over linear methods. The new nonlinear depth-inversion approach provides significant prospects for future practical applications in the field using existing remote sensing technologies, including continuous lidar scanners and stereo-imaging systems.

Plain Language Summary The coastal science community currently lacks insights into the morphological evolution of sandy beaches, including rapid changes that occur during storms. This is, to a large extent, explained by the difficulty to monitor the seabed elevation under such conditions in a region of the nearshore where high-energy waves break. If a relationship can be established between observed wave dynamics at the surface and the water depth below, remote-sensing technology presents a promising opportunity to reach this goal since it requires no physical interaction with the water environment. However, the existing algorithms to retrieve the water depth rely on the linear wave dispersion relation, which fails at describing the nonlinear dynamics of shoaling and breaking waves. Here, we develop a new depth-inversion approach based on a Boussinesq theory, which better describes such dynamics. Using a range of wave conditions and beach morphologies, we demonstrate that our approach results in significant improvement compared to the classic approaches, achieving typical accuracy within 10% in regions of the nearshore where waves break. The new nonlinear depth-inversion approach provides very promising prospects for future practical applications in the field using, for instance, high-resolution datasets collected with lidar scanners or stereo imaging systems.

1. Introduction

Understanding the temporal evolution of the nearshore bathymetry is critical to a wide range of applications including forecasting of coastal hazards, the morphological evolution of the sea/land interface and naval operations. However, mapping with sufficient accuracy and resolution the water depth along wave-dominated coastlines remains very challenging, especially in the region of energetic wave breaking in the surf zone. Remote-sensing technology, combined with depth-inversion algorithms, presents a promising opportunity to achieve this goal while minimizing risks associated with human intervention or the substantial challenges of installing and maintain in situ measurement equipment.

When currents are neglected, the linear wave dispersion relation provides a direct link between the spatial and temporal information of a surface wave field approaching the shore:

$$\omega^2 = g\kappa_L \tanh(\kappa_L h), \tag{1}$$

where $\omega = 2\pi f$ is the angular wave frequency, g is the acceleration of gravity, κ_L denotes the (single-valued) magnitude of the wavenumber vector $\vec{k}_L$ and h is the mean water depth. Depth-inversion algorithms such as *cBathy* (Holman et al., 2013) use this relationship (Equation 1) to infer depth from wave dispersive properties extracted from optical imagery (e.g., see Holman & Bergsma, 2021; Plant et al., 2008; Stockdon & Holman, 2000). In intermediate water depths, Equation 1 accurately describes the dispersive properties of low-amplitude wave fields so that typical errors on the water depth estimated with an algorithm like *cBathy* can be as low as 10% (e.g., see Brodie et al., 2018; Dugan et al., 2001; Holland, 2001). Closer to the breaking point and in surf zones, however, nonlinear amplitude dispersion effects intensify and significant deviations of dominant wavenumbers from the linear dispersion are expected (Elgar & Guza, 1985b; Herbers et al., 2002; Martins, Bonneton, & Michallet, 2021; Thornton & Guza, 1982). The present approaches based on optical imagery also suffer from inherent limitations due to the modulation transfer function, which relates the remotely-sensed wave properties to the real waveform (e.g., see Bergsma et al., 2019; Stockdon & Holman, 2000). These issues significantly affect the stability and accuracy of remotely-sensed wave dispersive properties, leading to errors on the water depths typically between 50% and 600% near and inside the surf zone (e.g., see Bergsma et al., 2016; Brodie et al., 2018; Catalán & Haller, 2008; Holland, 2001). New approaches are thus required in order to consistently reduce this error and bridge the existing gap in our capacity to map the topography-bathymetry continuum (Bergsma et al., 2021).

Technologies such as lidar scanners (Brodie et al., 2015; Fiedler et al., 2021; Martins et al., 2017) and stereo-video imagery (Bergamasco et al., 2017; de Vries et al., 2011) have seen major developments over the last decade and now allow the collection of accurate measurements of the sea-surface elevation in nearshore areas. By making information on wave heights directly accessible, these technologies offer the potential to substantially improve bathymetry inversion in the surf zone and right up to the shoreline. However, a universal nonlinear dispersion relation for shoaling and breaking waves is still lacking (for the most recent review refer: Catalán & Haller, 2008). Here, we describe a new depth-inversion method that relies on the stochastic Boussinesq theory of Herbers et al. (2002) to quantify nonlinear frequency and amplitude dispersion effects within both the shoaling and breaking wave regions. The new approach utilizes high-resolution datasets of free surface elevation and is designed so that it can be applied in the field with any technology collecting such data (e.g., lidar scanners, stereo-imagery systems). Suitable test datasets collected in the laboratory over both planar and barred beaches are used to demonstrate that the new nonlinear depth-inversion approach consistently outperforms the linear method (Equation 1), opening new perspectives for practical depth-inversion of surf zones in the field.

2. Methods

2.1. Experimental Data Sets

The new Boussinesq depth-inversion approach is developed and then evaluated using high-resolution surface elevation datasets collected in the laboratory. Here, the objective is to mimic under controlled conditions the field situation in which similar datasets can now be routinely collected using existing remote-sensing technologies. Though lidars presently offer the most robust and practical solution for collecting highly-resolved surface elevation data in the field, the approach presented is applicable to any technology capable of collecting such data (e.g., stereo imagery systems).

We consider three specific series of experiments, which covered a relatively wide range of wave conditions and beach morphologies. The experiments of van Noorloos (2003) were performed over a 1:35 planar beach in the 40 m-long wave flume at Delft University of Technology (Figure 1; see also van Dongeren et al., 2007). A second planar beach case originates from the Gently sLOping Beach Experiment (GLOBEX) performed over a mildly-sloping concrete beach (1:80) specifically built in a 110 m-long wave flume in Delft, the Netherlands (Figure 1; see also Ruessink et al., 2013). Finally, we use a 30 min-long sequence extracted from the experiments performed over a mobile bottom in the 36 m-long LEGI flume and described in Michallet et al. (2011). The sediment for this latter experiment was chosen such that the Shields and Rouse numbers were of similar magnitude as those found in natural environments (Grasso et al., 2009). The beach profile exhibited a pronounced sandbar that migrated landward by about 2.5 m during the wave sequence (Figure 1).

For the planar beach cases, we concentrate on the most energetic tests performed with irregular waves. For the experiments of van Noorloos (2003), this corresponds to the C_3 wave test, characterized by a significant wave height $H_{m0} = 0.1$ m and peak frequency $f_p = 0.5$ Hz. For GLOBEX, this corresponds to the A2 wave test ($H_{m0} = 0.2$ m; $f_p = 0.444$ Hz). During the experiments of Michallet et al. (2011), the conditions consisted of irregular waves characterized by $H_{m0} = 0.16$ m and $f_p = 0.4$ Hz. The free surface elevation ζ was collected at high spatial resolution, which generally varied across the direction of wave propagation (Figure 1).

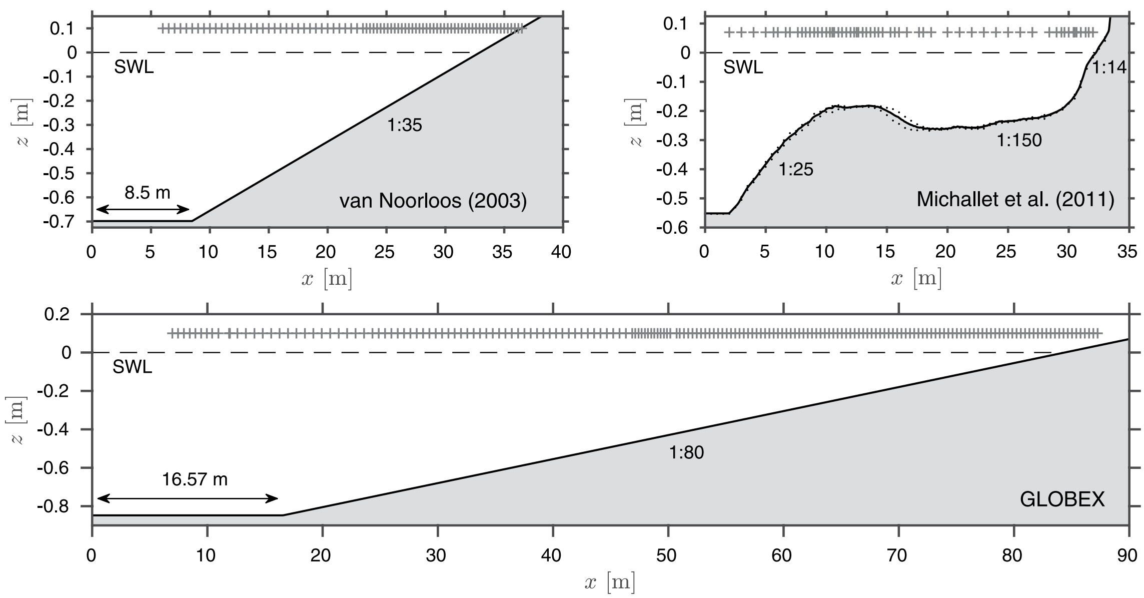

Figure 1. Beach elevation z against the cross-shore distance x for the experiments of van Noorloos (2003, top left), Michallet et al. (2011, top right) and GLOBEX (Ruessink et al., 2013, bottom). The wave paddle is located at $x = 0$ m and gray "+" symbols show the wave gauges location. The barred beach profile for the experiments of Michallet et al. (2011) was obtained by averaging the elevations measured before and after the wave sequence, which are shown as black dotted lines (most morphological changes concentrate over the bar, $x = 7 - 18$ m).

2.2. Estimating and Predicting Wave Dominant Dispersive Properties

In the nearshore region, nonlinear interactions between triads of frequencies lead to the growth of forced high-frequency components (Elgar & Guza, 1985a; Freilich et al., 1984; Herbers et al., 2000; Phillips, 1960). Both free and forced wave components then co-exist at a given frequency, causing deviations of dominant wavenumbers from the linear wave dispersion relation (Elgar & Guza, 1985b; Herbers et al., 2002; Martins, Bonneton, & Michallet, 2021). In practice, dominant wavenumber spectra are estimated from cross-spectral analyses between adjacent pressure (Elgar & Guza, 1985b; Herbers et al., 2002) or resistance-type wave gauges (Martins, Bonneton, & Michallet, 2021). In the present 1D configuration, we follow the procedure described in Martins, Bonneton, and Michallet (2021) to estimate the dominant wavenumber spectra κ_{obs} across the experiments. A maximum distance of $0.3L_p$ was allowed between wave gauges for the cross-spectral analysis, where L_p is the peak wavelength predicted by the linear wave dispersion relation (Equation 1).

Dominant wavenumber spectra κ_{rms} are then estimated from the surface elevation ζ using the Boussinesq theory of Herbers et al. (2002):

$$\kappa_{rms}(\omega) = \frac{\omega}{\sqrt{gh}} \sqrt{1 + h\gamma_{fr,1}(\omega) + h^2\gamma_{fr,2}(\omega) - \frac{1}{h}\gamma_{am}(\omega)}, \tag{2}$$

with

$$\gamma_{fr,1}(\omega) = \frac{\omega^2}{3g} \tag{3}$$

$$\gamma_{fr,2}(\omega) = \frac{\omega^4}{36g^2} \tag{4}$$

$$\gamma_{am}(\omega) = \frac{3}{2E(\omega)} \int_{-\infty}^{\infty} \mathrm{Re}\left\{B\left(\omega', \omega - \omega'\right)\right\} d\omega', \tag{5}$$

where E and B are the spectral and bispectral densities of ζ respectively, and Re{.} denotes the real part. Further details on the computation of cross-spectral, spectral and bispectral estimates are provided in the Supporting Information S1. In Equation 2, the leading-order term corresponds to the wavenumber for non-dispersive shallow-water waves. Terms with $\gamma_{fr,1}$ and $\gamma_{fr,2}$ are second and fourth-order frequency dispersion terms, respectively, while γ_{am} is a second-order amplitude dispersion term. Compared to the original expression for κ_{rms} given by Herbers et al. (2002, their Equation 12), we kept the fourth-order frequency term $\gamma_{fr,2}$ in order to improve the linear dispersive properties of the Boussinesq approximation. Each term was also here expressed in a way that h remains isolated, which facilitates the depth-inversion procedure (Section 2.3).

The Boussinesq approximation of κ_{rms} (Equation 2) was derived assuming that the wave field is weakly nonlinear, weakly dispersive, and that these effects are of similar order. By introducing the dispersive term $\mu = (\kappa_p h)^2$, in which κ_p is the peak wavenumber given by the linear dispersion relation, and the amplitude term $\epsilon = H_{m0}/2h$, this corresponds to Ursell numbers $U_r = \epsilon/\mu$ around unity. In the following, we will only consider regions of the wave flumes where $U_r \gtrsim 0.3$. Herbers et al. (2002) further assume the water depth to be slowly varying with regards to the scale of nonlinear energy exchanges. In our current notation, this writes $\beta/\sqrt{\mu} \ll 1$, where β is a characteristic bottom slope. For the cases investigated here, $\beta/\sqrt{\mu} < 0.1$, except near the shoreline for the cases of van Noorloos (2003) and Michallet et al. (2011), where it reaches 0.17–0.18 at most. Thus, this hypothesis is here always verified, even over the steep sandbar characterizing the experiments of Michallet et al. (2011).

2.3. Depth-Inversion Procedure

The new depth-inversion procedure relies on the capacity of the Boussinesq theory of Herbers et al. (2002) to accurately predict the dominant wavenumbers across the shoaling and breaking wave regions (Herbers et al., 2002; Martins, Bonneton, Lannes, & Michallet, 2021). When the free surface elevation is measured, the mean water depth h is the only unknown in Equations 2–5. At each cross-shore location, h can then be retrieved through a minimization problem, based on the match between observed κ_{obs} and predicted κ_{rms} spectra.

The mean water depth at each observation location corresponds to the depth h that minimizes the following expression:

$$\sum_{\omega_i=\omega_{\min}}^{\omega_{\max}} \alpha_i \left(\kappa_{obs}(\omega_i) - \kappa_{rms}(\omega_i) \right)^2 = \sum_{\omega_i=\omega_{\min}}^{\omega_{\max}} \alpha_i \left(\kappa_{obs}(\omega_i) - \frac{\omega_i}{\sqrt{gh}} \sqrt{1 + h\gamma_{fr,1}(\omega_i) + h^2\gamma_{fr,2}(\omega_i) - \frac{1}{h}\gamma_{am}(\omega_i)} \right)^2, \quad (6)$$

where α_i are weights and $[\omega_{\min}; \omega_{\max}]$ defines the frequency range over which the minimization is performed. Though the water depth estimates in the present study were found to be relatively insensitive to the use of frequency-dependent weights, we used the coherence obtained from the cross-spectral analyses employed to estimate κ_{obs}. In the following, we consider the range of frequencies $[0.7\omega_p; 2.5\omega_p]$, which includes the principal components (corresponding to sea/swell) and their first harmonic. This upper limit approximately corresponds to the frequency where the Boussinesq theory of Herbers et al. (2002) starts to decrease in accuracy within the nearshore region (see also Martins, Bonneton, Lannes, & Michallet, 2021).

The mean water depth estimated with the Boussinesq theory of Herbers et al. (2002) is compared with estimates from the linear wave dispersion relation (Equation 1), which minimizes the following expression:

$$\sum_{\omega_i=\omega_{\min}}^{\omega_{\max}} \alpha_i \left(h - \frac{1}{\kappa_{obs}(\omega_i)} \tanh^{-1}\left[\frac{\omega_i^2}{\kappa_{obs}(\omega_i) g} \right] \right)^2 \quad (7)$$

3. Results

3.1. Assessment of the Boussinesq Theory for Estimating Nearshore Wave Dispersive Properties

Prior to testing the new nonlinear depth-inversion approach, we first assess the capacity of the Boussinesq theory (Equation 2) to predict the dispersive properties of irregular waves in both shoaling and breaking conditions. Figure 2 shows the cross-shore evolution of observed and predicted dominant wave phase velocity $c(\omega) = \omega/\kappa(\omega)$ at the peak frequency ω_p (Figures 2g–2i) and second harmonic $2\omega_p$ (Figures 2j–2l). The significant wave height (Figures 2a–2c), as well as dispersive μ and amplitude ϵ parameters (Figures 2d–2f), are also shown since they are good indicators of the relative position in the flumes (i.e., the presence of shoaling/breaking waves). In all tests considered here, wave breaking occurs for $U_r = \epsilon/\mu \sim 1$.

The Boussinesq theory of Herbers et al. (2002) accurately predicts the cross-shore evolution of dominant wave phase velocity at both the peak frequency ω_p (Figures 2g–2i) and the second harmonic $2\omega_p$ (Figures 2j–2l). This

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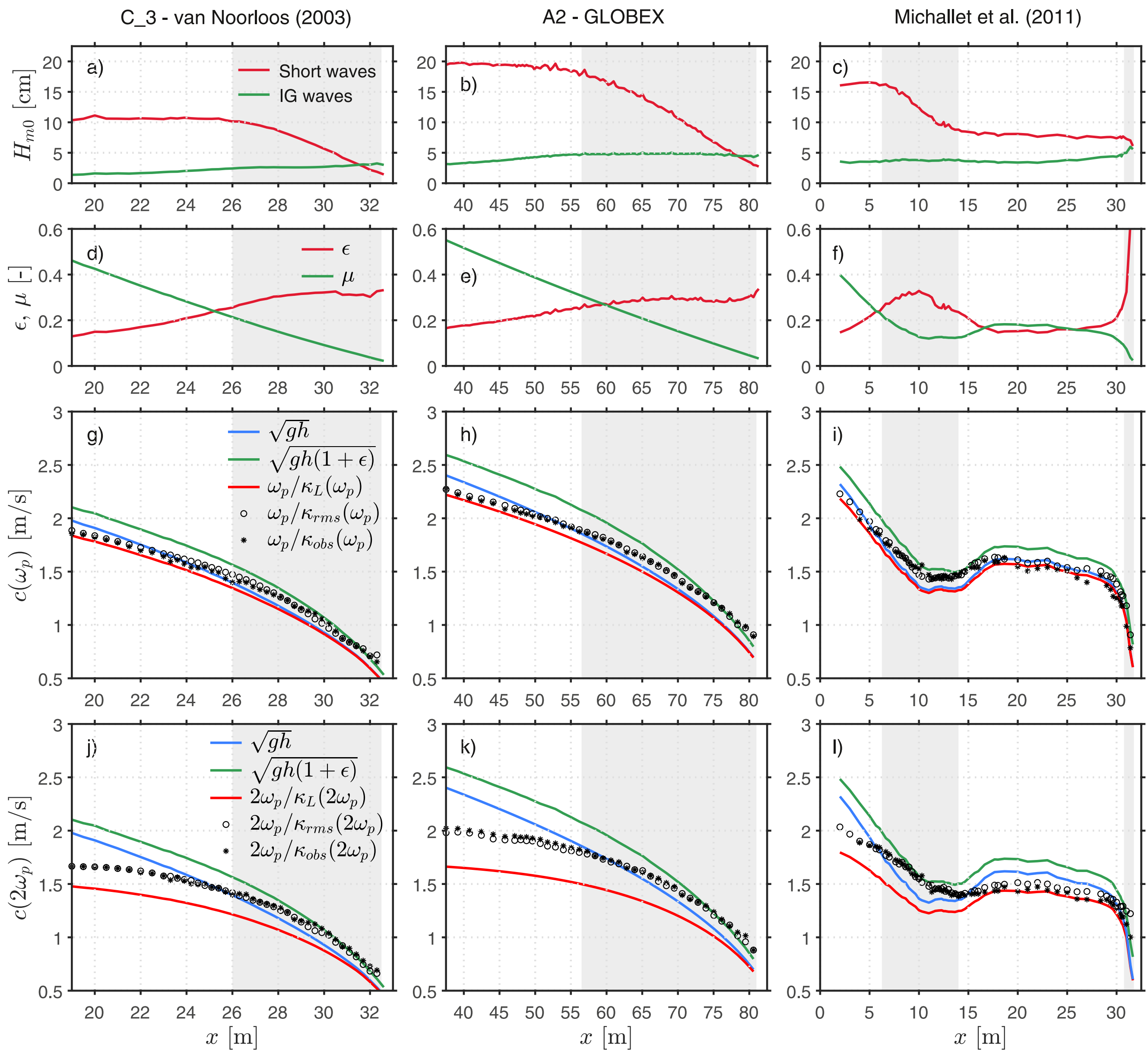

Figure 2. Assessment of the Boussinesq theory (Equation 2) to predict the cross-shore evolution of dispersive properties during the experiments of van Noorloos (2003, left panels), GLOBEX (middle panels) and Michallet et al. (2011, right panels). Panels (a–c) show the cross-shore evolution of significant wave height H_{m0} for short and infragravity (IG) waves computed as $\left(16\,\overline{\zeta^2}\right)^{1/2}$ (cutoff frequency at $0.6f_p$). Panel (d–f) show the amplitude ($\epsilon = H_{m0}/2h$) and dispersion ($\mu = \left(\kappa_p h\right)^2$) parameters. Panels (g–j) show the observed and Boussinesq predictions of the wave phase velocity at the peak frequency ω_p, while panels (k–m) show those at the second harmonic $2\omega_p$. These quantities are compared with the predictions from the linear wave dispersion (Equation 1) and shallow-water predictors. In all panels, the gray shaded area indicates regions of the wave flume where wave breaking occurs.

confirms that the theory accurately quantifies the variation of nonlinear amplitude dispersion effects across both the shoaling region and the surf zone. At the peak frequency, deviations of observed wave phase velocities from the linear predictions steadily increase as short waves approach the breaking point and the maximum of these deviations is reached close to the shoreline for both planar beaches (up to 30% differences, see Figures 2g and 2h). For the barred beach, this occurs on the landward edge of the sandbar ($x \sim 14$ m), corresponding to a 10% difference (Figure 2j). At $2\omega_p$, nonlinear energy transfers between triads of frequencies (mostly self-interactions around ω_p) explain the large deviations from the linear prediction deep in the shoaling region. For the two planar beaches (Figures 2j and 2k), these deviations reach their maximum at locations corresponding to $U_r = \epsilon/\mu \sim 0.3$–$0.4$ and remain quite steady across both the shoaling region and surf zone (15%–20% differences for both datasets). For the

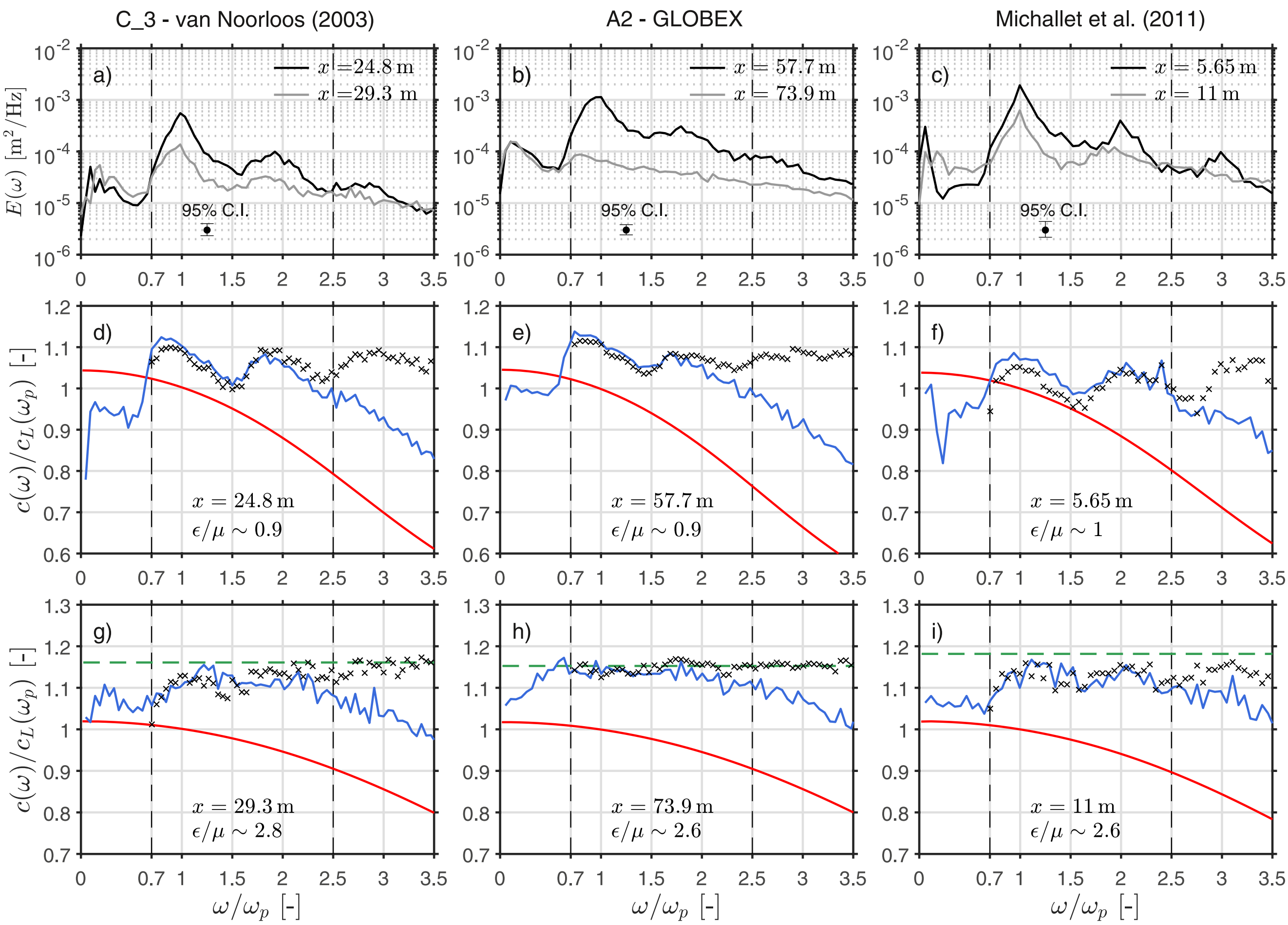

Figure 3. Assessment of the Boussinesq theory (Equation 2) to predict wave phase spectra for the experiments of van Noorloos (2003, left panels), GLOBEX (middle panels) and Michallet et al. (2011, right panels). Panels (a–c) show the energy density spectra of ζ at two positions corresponding to shoaling panels (d–f) and breaking situations panels (g–i). The normalized wave phase velocities predicted with the Boussinesq (blue lines) and linear wave (red line) theories are compared against observations (black crosses). In the surf zone panels (g–i), the green horizontal line corresponds to the modified shallow-water wave celerity predictor ($\sqrt{gh(1+\epsilon)}$). The cross-shore locations were selected based on the Ursell number ($U_r \sim 1$ and $U_r \sim 2.6$–2.8 for shoaling and breaking situations, respectively) and are indicated for each experiment. The vertical lines indicate the range of frequencies [$0.7\omega_p$; $2.5\omega_p$] used for the depth-inversion.

barred case, these differences reach 25% above the sandbar, where wave breaking is most intense ($x = 9 - 10$ m, see Figure 2l).

Figure 3 shows that the accuracy of the Boussinesq theory extends across the whole range of frequencies [$0.7\omega_p$; $2.5\omega_p$], which is consistent with the results of Herbers et al. (2002) and Martins, Bonneton, Lannes, and Michallet (2021). Two examples taken from the shoaling region close to the breaking point ($U_r \sim 1$) and in the surf zone ($U_r \sim 2.6$–2.8) are shown in Figures 3d–3f and 3g–3i, respectively. As discussed in Martins, Bonneton, and Michallet (2021) for the GLOBEX case, the deviations of observed wave phase velocity spectra from linear predictions at a given frequency ω increase with the intensity of nonlinear energy transfers and the relative amount of forced energy at ω. Together with the spectral bandwidth of incident short waves (Figures 3a–3c), this explains the frequency-dependence of deviations from linear predictions observed in the shoaling region (Figures 3e–3g). In the surf zone, most components travel almost at the same velocity (Elgar & Guza, 1985b; Martins, Bonneton, & Michallet, 2021; Thornton & Guza, 1982), which explains the relatively constant observed wave phase velocity across all frequencies (Figures 3g–3i). Overall, the Boussinesq theory of Herbers et al. (2002) accurately describes the dynamics of wave fields in both shoaling and surf zone situations. For all experiments, a slight positively bias can be noted in Boussinesq predictions at frequencies corresponding to the most energetic components (up to 3%–4% difference between [$0.7\omega_p$; $1.5\omega_p$], see Figures 3d–3f). This overestimation appears quite consistent across the shoaling region for the two planar cases (Figures 2g and 2h).

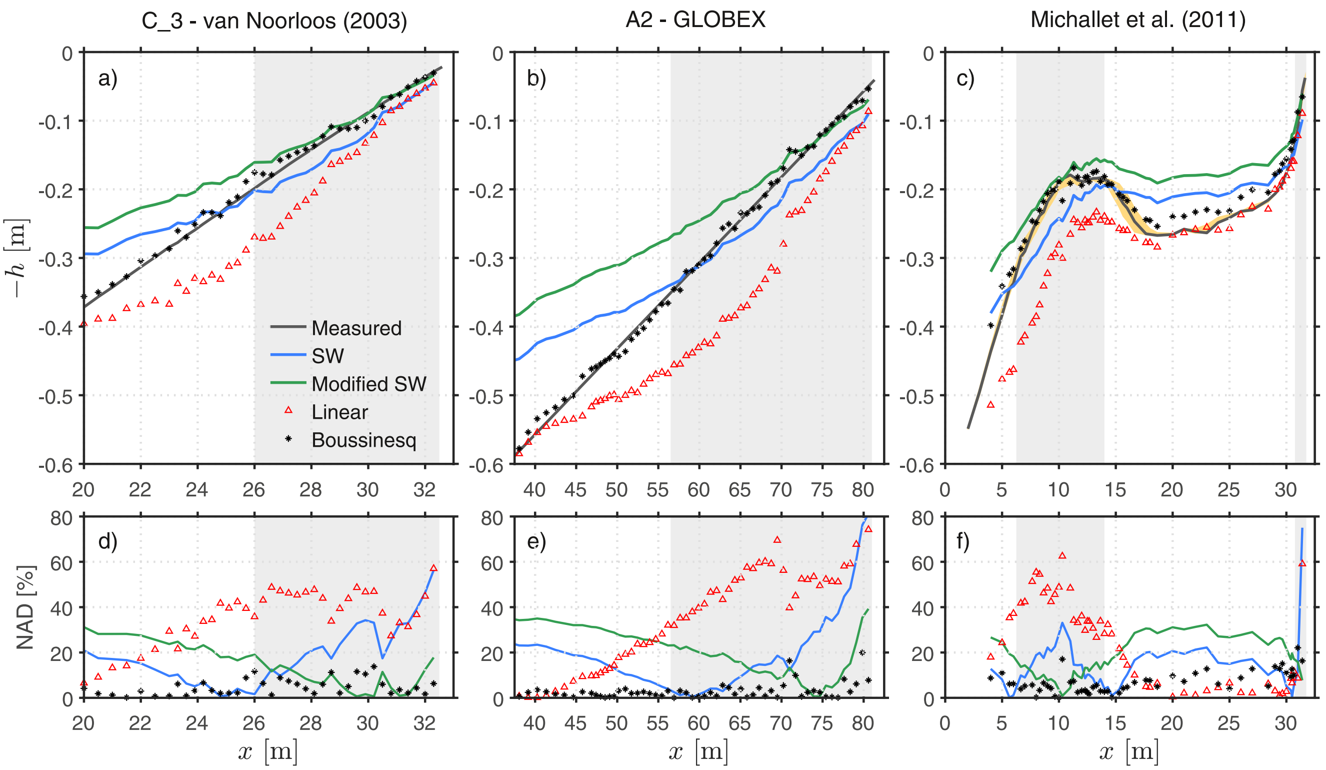

Figure 4. Results of the depth inversion applications for the experiments of van Noorloos (2003, left panels), GLOBEX (middle panels) and Michallet et al. (2011, right panels). Panels (a–c) show the beach elevation profile estimated using Boussinesq (Equation 6) and the linear wave theory (Equation 7). These are compared with estimates based on shallow-water waves propagation velocity ("SW": $c_{bulk} \sim \sqrt{gh}$ and "Modified SW": $c_{bulk} \sim \sqrt{gh(1+\epsilon)}$). In panel (c), the orange-shaded area around the measured profile corresponds to the bed elevation changes observed during the considered wave sequence. Panels (d–f) show the corresponding normalized absolute difference (NAD) of measured and predicted water depths. In all panels, the gray shaded area indicates regions of the wave flume where wave breaking occurs.

3.2. Depth-Inversion Applications

Boussinesq (Equation 6) and linear (Equation 7) estimates of the mean water depth h are shown in Figure 4. These are compared against estimates obtained assuming that all incident waves propagate as fast as shallow-water waves $\left(c_{bulk} \sim \sqrt{gh}\right)$ or slightly faster, due to nonlinear amplitude effects $\left(c_{bulk} \sim \sqrt{gh(1+\epsilon)}\right)$. The bulk wave celerity c_{bulk} is computed through simple cross-correlation between two wave gauges (e.g., Martins et al., 2016; Tissier et al., 2011).

In both the shoaling region and the surf zone, the new Boussinesq approach substantially improves the water depth predictions compared to the linear method. For the C_3 wave test of van Noorloos (2003), the normalized error associated with the Boussinesq approach remains small (<10%), except at the early stage of the surf zone ($x = 25 - 29$ m, see Figures 4a and 4d). The error is generally <5% for the most nonlinear test of GLOBEX (Figures 4b and 4e), except at a few locations in the surf zone where it reaches ∼10% (20% locally). This strongly contrasts with the increasing error of the linear method, which overestimates the mean water depth by over 40% across the surf zone of the planar beaches considered here. The overestimation reaches up to 80% near the shoreline for the GLOBEX case (Figures 4b and 4e). The Boussinesq approach also performs well in the barred beach case (Figures 4c and 4f), especially around the sandbar where mean water depths are estimated within 10% (compared to a ∼40 − 60% overestimation with the linear approach). It is interesting to note that the beach trough section ($x = 17 - 28$ m, Figure 4c) corresponds to the only region for all three experiments where the linear approach outperforms the new Boussinesq approach. This is explained by the release of bound high-harmonics as short waves leave the sandbar region, a phenomenon already reported and described in the literature (e.g., see Becq-Girard et al., 1999; Beji & Battjes, 1993; Masselink, 1998). In terms of wave phase velocity, this is evidenced in the close match between the observations and predictions by the linear wave dispersion at both the peak frequency (Figure 2i) and the second harmonic (Figure 2l).

Consistent with the large discrepancies between $\sqrt{gh}$ and the observed wave phase velocities for all experiments (Figures 2g–2l), the linear-based shallow-water predictor $\left(c_{bulk} \sim \sqrt{gh}\right)$ poorly performs across both the shoaling and breaking regions considered here. Though the modified shallow water-based predictor $\left(c_{bulk} \sim \sqrt{gh(1+\epsilon)}\right)$ has been observed to improve the prediction of wave phase velocities in inner surf zones (Martins, Bonneton, & Michallet, 2021; Martins et al., 2018; Tissier et al., 2011), its performance here is quite mixed. For the C_3 wave test of van Noorloos (2003), the error made on h is of similar order as the proposed Boussinesq approach, except very close to the shoreline where it reaches 20% (Figure 4d). The performances substantially deteriorate for the A2 test during GLOBEX, where the error remains high over a large portion of the surf zone and reaches up to 40% near the shoreline (Figure 4e). For the barred beach case (Figures 4c and 4f), the error remains high everywhere (∼30%), except above the sandbar where nonlinear effects are strongest (Figure 2f).

4. Discussion and Concluding Remarks

Developing the capacity to map nearshore and surf zone bathymetry right up to the shoreline is a prerequisite to accurately quantify the morphological evolution of sandy beaches. Depth-inversion algorithms applied to remotely-sensed surface wave properties are a very promising approach to achieving this goal. However, present solutions often incorporate technical and/or theoretical limitations, which limit their accuracy in the breaking wave region. Here, we present and test a new depth-inversion approach based on the stochastic Boussinesq theory of Herbers et al. (2002) that is applied to high-resolution maps of free surface elevation. As for most depth-inversion algorithms, the error made on the water depth estimates has two principal sources: (a) observed $\omega - \kappa$ pairs, whose accuracy very much depends on the nature of the data; and (b) the theoretical framework for retrieving depth from those observations. Using surface elevation datasets, which can be collected in the field with lidar scanners, first has for benefit to reduce potentially large errors in the estimation of wavenumbers from optical imagery due to the modulation transfer function (Bergsma et al., 2019; Stockdon & Holman, 2000). Using a Boussinesq theoretical framework on surface elevation datasets then allows to accurately predict both nonlinear frequency and amplitude dispersion effects, overcoming the limitations of the linear wave dispersion in regions of the nearshore where nonlinear effects strongly alter the dispersive properties of incident waves (Elgar & Guza, 1985b; Herbers et al., 2002; Martins, Bonneton, & Michallet, 2021; Thornton & Guza, 1982).

For the relatively wide range of wave conditions and beach morphologies considered herein, the proposed Boussinesq approach results in enhanced levels of accuracy in the surf zone. Boussinesq estimates of the mean water depth are typically accurate within 10%, which substantially improves the predictions compared to the linear wave dispersion relation (errors in the range 40%–80% across the surf zone). Considering frequencies just around the energy peak $[0.7\omega_p; 1.5\omega_p]$ during the minimization procedure (Equation 7) typically halves the error made in both the shoaling and breaking wave regions (see Figure S3 in Supporting Information S1), though an 80% overestimation is still obtained at the shoreline during GLOBEX. Given that discrepancies between observed wave phase speeds and those predicted by the linear dispersion relation are minimal around the peak frequency, this corresponds to the smallest error that can be reached in the surf zone with a linear-based approach. In contrast, the range of frequencies considered here only has a limited impact on the performances of the Boussinesq approach, which is explained by the accuracy of the theory at least up to $2.5\omega_p$ (Figures 3d–3i).

Until now, we have principally investigated the sensitivity of depth estimates to the theoretical framework used during the depth-inversion phase. However, as mentioned above, errors in final depth estimates can also originate from the procedure to extract the wave dispersive properties. Here, the main source of uncertainty on wavenumber estimates is thought to be related to the time-synchronization of wave gauges. Imprecise time-synchronization procedures introduce time lags related to the sampling frequency f_s (maximal lag is $0.5/f_s$), resulting in errors in wavenumber estimates. We estimated that such procedures could, at most, lead to 3% errors during GLOBEX and the experiments of Michallet et al. (2011) (see Figure S2 in Supporting Information S1). For the observations reported by van Noorloos (2003), the potential errors reach 10%, which is consistent with the larger errors on water depths obtained for these particular experiments. In typical field situations, where all data are collected simultaneously, this source of error can be avoided. In the proposed Boussinesq approach, an additional source of error originates from the estimation of the nonlinear amplitude dispersion term γ_{am}. By analyzing the sensitivity of depth estimates to varying levels of noise in the input signal (Figure S1 in Supporting Information S1), it was found that γ_{am} is relatively insensitive to levels of noise that are realistic for surface elevation datasets collected in the field, for example, by lidars. The systematic noise in lidar data is typically two orders of magnitude lower than incident wave amplitudes, so that negligible influence of noise on the mean water depth estimates is expected.

Though bulk wave celerity can be easily estimated at large spatial scales from optical imagery in the field (e.g., Lippmann & Holman, 1991), the new work presented here has highlighted the limitations of shallow-water waves predictor $\left(c_{bulk} \sim \sqrt{gh}\right)$ for local depth-inversion applications. The modified predictor $\left(c_{bulk} \sim \sqrt{gh(1+\epsilon)}\right)$ empirically incorporates nonlinear amplitude effects and leads to improved water depths estimation in inner surf zones, however, two main issues arise with this predictor: the accuracy appears limited under highly nonlinear conditions (Figures 4e and 4f), and the seaward boundary limit where it can be used remains uncertain. Limited accuracy is thus expected when a wide range of incident wave conditions and/or beach morphology is considered. The new Boussinesq approach does not suffer from these limitations, mainly because it accurately predicts both frequency and amplitude nonlinear dispersion effects. Importantly, the proposed approach does not require any form of calibration, thus laying the basis for a universal depth-inversion relationship for nearshore and surf zone regions. The development of this new method was motivated by the recent widespread collection of high-resolution free surface elevation datasets by lidar scanners in the field (e.g., Brodie et al., 2015; Martins et al., 2018; Fiedler et al., 2021). Lidar scanners have the unique feature that they directly measure both surf zone processes and the subaerial section of sandy beaches. In combination with the proposed nonlinear depth-inversion procedure, these sensors open a whole new range of possibilities for continuous monitoring of the morphological evolution of sandy beaches extending from the nearshore to the dunes.

Data Availability Statement

The raw data from GLOBEX used in this research can be accessed on Zenodo at https://doi.org/10.5281/zenodo.4009405 and can be used under the Creative Commons Attribution 4.0 International license. All the processed data and software produced in this study are available at https://doi.org/10.5281/zenodo.7511943.

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