

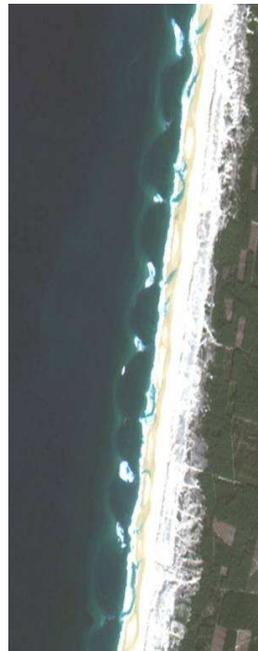
**Alongshore differential topographically controlled
wave-breaking and rip current circulation**

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<http://www.epoc.u-bordeaux.fr/methys/>



Wave-induced 2DH circulation

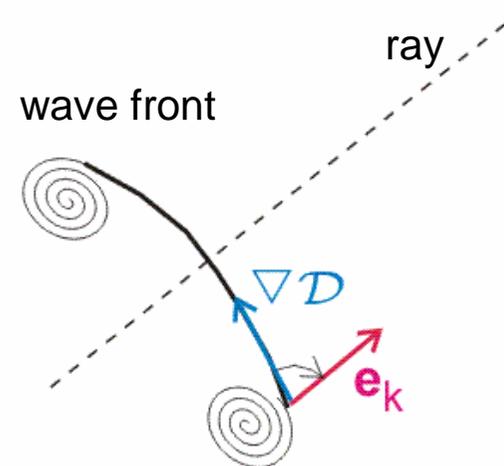
- ❑ **2D horizontal circulation ↔ vertical vorticity field**

- ❑ **2DH wave-averaged approach**
 - ⇒ generation of mean flow vertical vorticity is an old problem (see *Bowen (1969)*) but still an open one
 - ⇒ improvement of 2DH circulation modeling is a necessary step for the development of more complex quasi-3D (e.g. *Haas et al. (2003)*) or 3D (e.g. *Mellor (2003)*, *Ardhuin (2008)*, *Reniers et al. (2008)*) approaches.

Wave-induced vertical vorticity

Vorticity generation by a bore in shallow water

NSWE theory \Rightarrow non-uniformities along the breaking-wave crest drive vertical vorticity, *Peregrine (1998)*



For 2DH wave-averaged approaches, what is the resulting effect of this mechanism integrated over a wave period ?

2DH mean current vorticity \Rightarrow Bowen (1969) a pioneering work

$$\frac{\partial \mathbf{U}_T}{\partial t} + (\mathbf{U}_T \cdot \nabla) \mathbf{U}_T + g \nabla \bar{\zeta} = \mathbf{R} + \mathbf{V}_s + \mathbf{F}_r$$

$\mathbf{U}_T = \frac{1}{h} \overline{\int_{-d}^{\zeta} \mathbf{u}_H dz}$: mean transport horizontal velocity

$$R_i = -\frac{1}{h} \frac{\partial S_{ij}}{\partial x_j}$$

$$\frac{\partial \omega_T}{\partial t} + \nabla \cdot (\omega_T \mathbf{U}_T) = \nabla \wedge \mathbf{R} + \nabla \wedge \mathbf{V}_s + \nabla \wedge \mathbf{F}_r$$

Outside the surf zone: $\nabla \wedge \mathbf{R} = 0$.

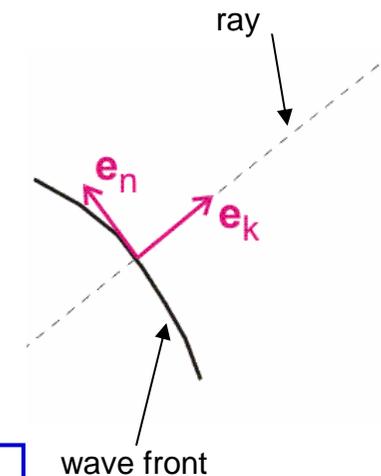
Inside the surf zone: the forcing term $\nabla \wedge \mathbf{R}$ is implicitly related to wave energy dissipation (see *Longuet-Higgins and Stewart 1973* and *Battjes 1988*).

Reformulation of the mean momentum equation: *Smith (2006)*

$$\bar{h}\mathbf{U}_T = \overline{\int_{-d}^{\zeta} \mathbf{u}_H dz} = \bar{h}\mathbf{U} + \underbrace{\int_{-d}^{\zeta} \tilde{\mathbf{u}}_H dz}_{\tilde{\mathbf{M}}}$$

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_j} (A(c_{gj} + U_j)) = -\frac{D_{bm}}{\sigma}$$

$$A = E/\sigma \quad \tilde{M}_i = Ak_i$$



$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} + g \nabla \bar{\zeta} = \mathcal{D} \mathbf{e}_k - \nabla \tilde{J} + \frac{\tilde{\mathbf{M}}}{\bar{h}} \wedge (\nabla \wedge \mathbf{U}) + \mathbf{V}_s + \mathbf{F}_r$$

$$\tilde{J} = E \frac{k}{\sinh(2k\bar{h})} \quad \mathcal{D} = \frac{D_{bm}}{\bar{h}c_\phi}$$

Vorticity equation for the wave-induced circulation: *Bonneton et al (DCDS-S 2010)*



$$\frac{\partial \omega}{\partial t} + \nabla \cdot \left(\omega \left(\mathbf{U} + \frac{\tilde{\mathbf{M}}}{\bar{h}} \right) \right) = \underbrace{\nabla \wedge (\mathcal{D} \mathbf{e}_k)}_{\sim \nabla \mathcal{D} \wedge \mathbf{e}_k} + \nabla \wedge \mathbf{V}_s + \nabla \wedge \mathbf{F}_r$$

$$\mathcal{D} = \frac{D_{bm}}{\bar{h} c_\phi}$$

vorticity forcing term

Remark : the gradient of mean elevation (setup/setdown)

is not a driving force for the circulation

but:

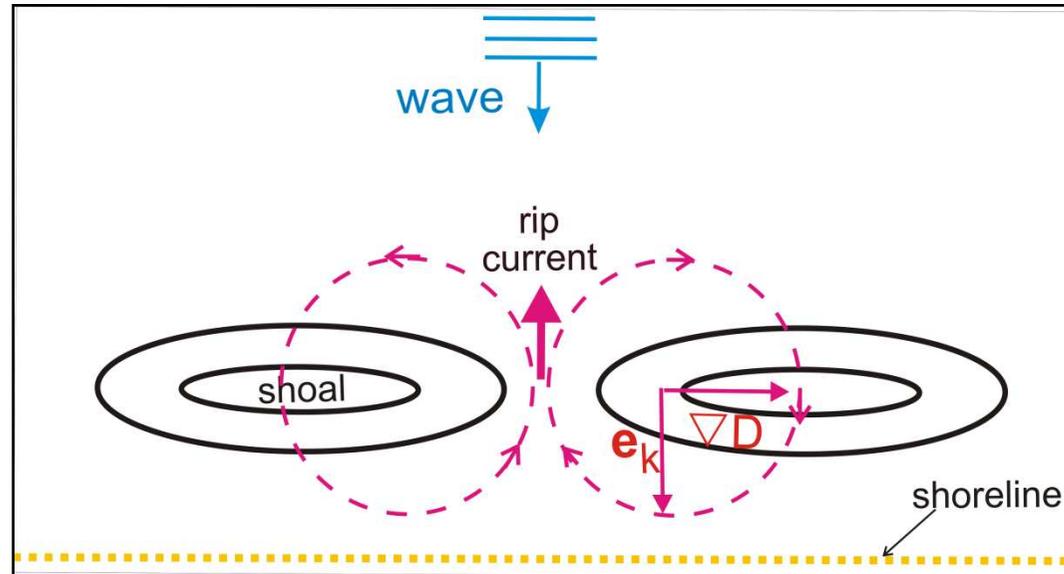
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$$\mathcal{D} = \frac{D_{bm}}{\bar{h} c_\phi}$$



Vorticity equation for a stationary wave forcing

$$\frac{\partial \omega}{\partial t} + \nabla \cdot \left(\omega \left(\mathbf{U} + \frac{\tilde{\mathbf{M}}}{\bar{h}} \right) \right) = \nabla \mathcal{D} \wedge \mathbf{e}_k + \nabla \wedge \mathbf{V}_s + \nabla \wedge \mathbf{F}_r$$

$$V_{s_i} = \frac{1}{\bar{h}} \frac{\partial}{\partial x_j} \left(\nu_t \bar{h} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right)$$

$$\nabla \mathcal{D} \wedge \mathbf{e}_k + \underbrace{\nabla \wedge \mathbf{V}_s}_{\sim \nu_t \nabla^2 \omega} \simeq 0$$

\mathcal{D} and ν_t are the key parameters

- 2DH SWAN/MARS coupling model based on

Smith's equations: *Bruneau (2009, PhD.)*

classical parametrisations:

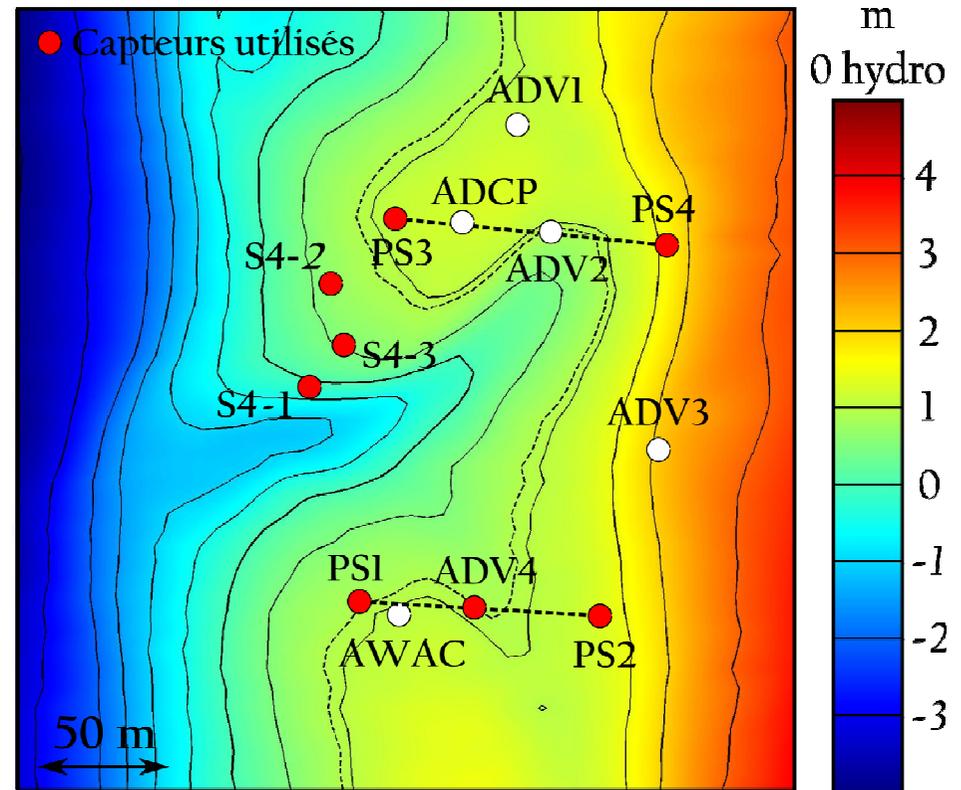
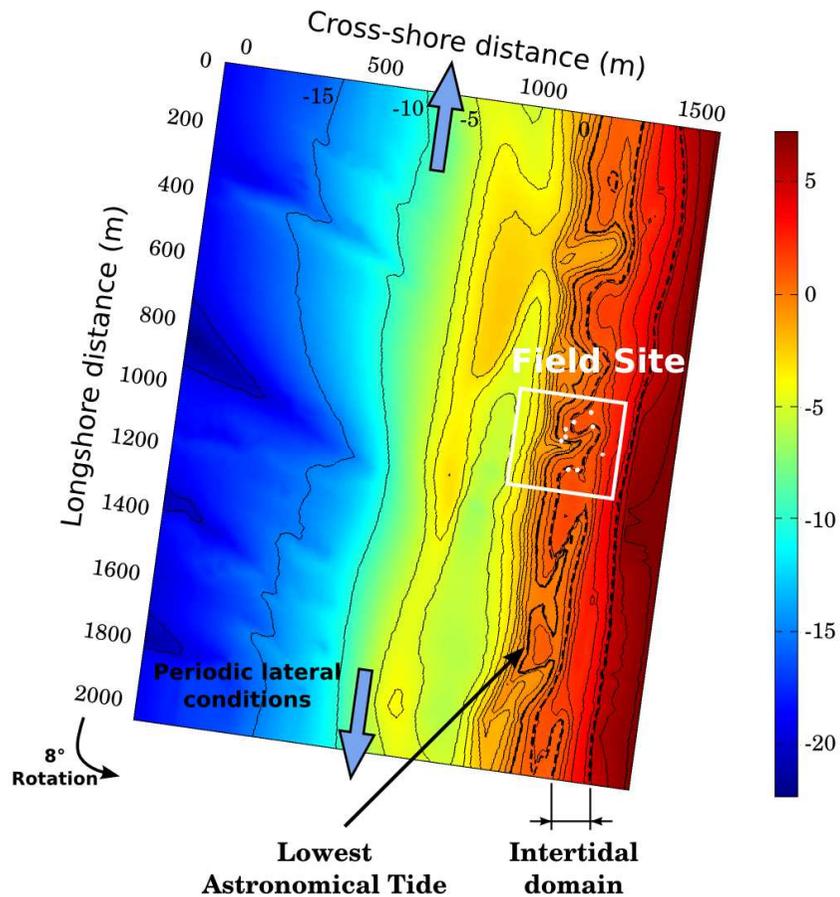
$\mathcal{D} \Rightarrow$ Battjes and Janssen (1978) $v_t \Rightarrow$ Battjes (1975)

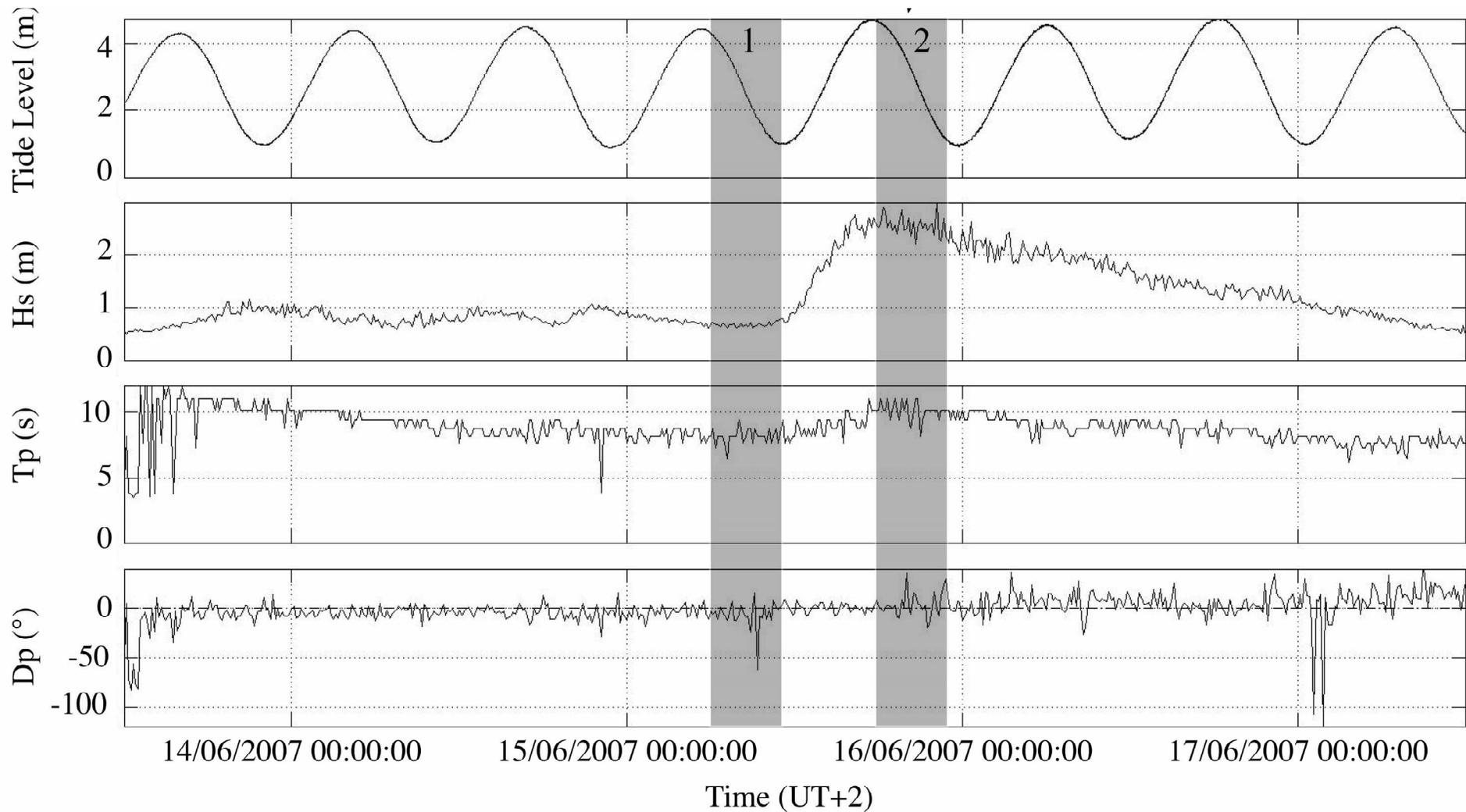
- comparisons with field measurements

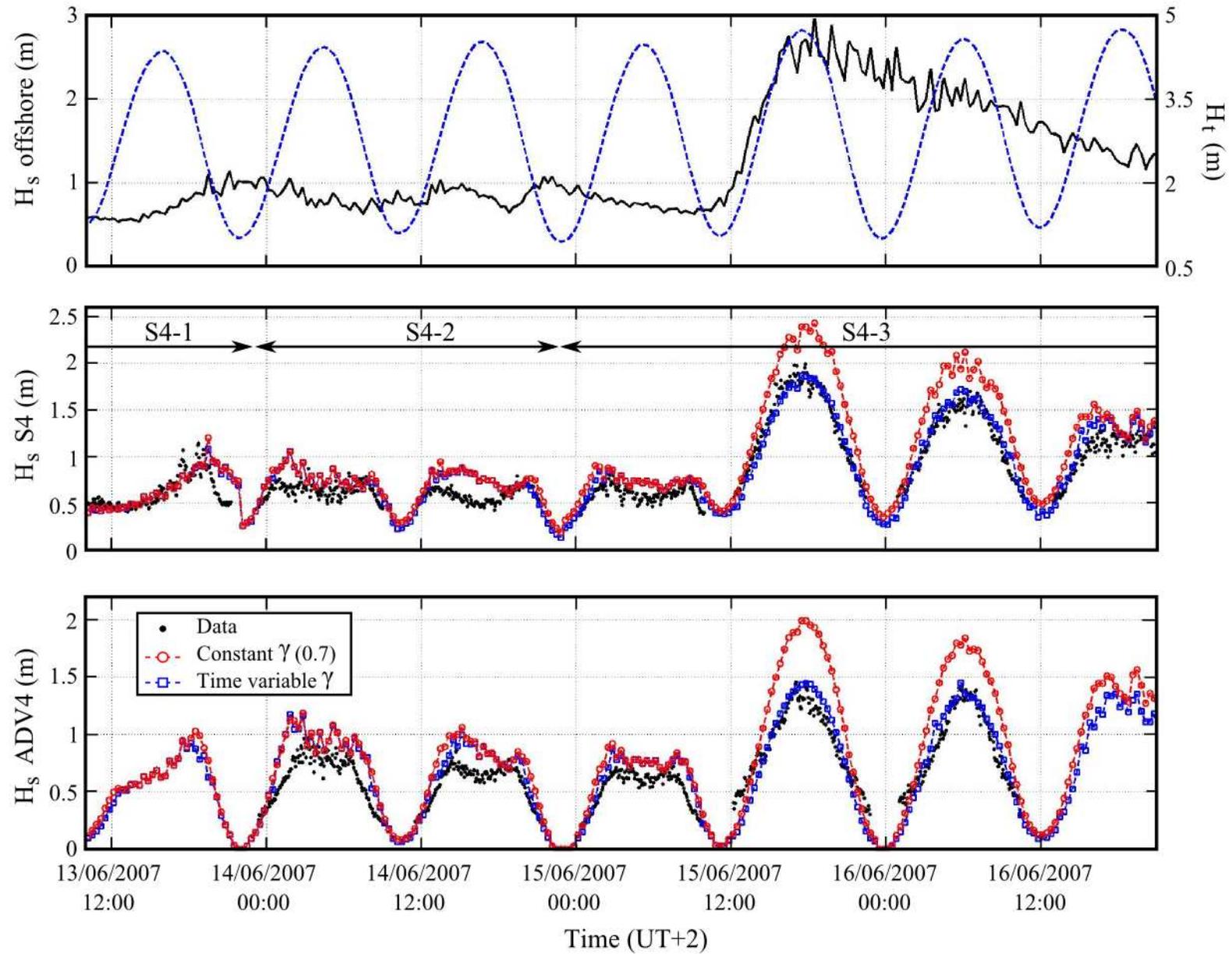
Biscarosse Beach 2007: *Bruneau et al (2009, CSR)*

Wave-induced circulation

Applications

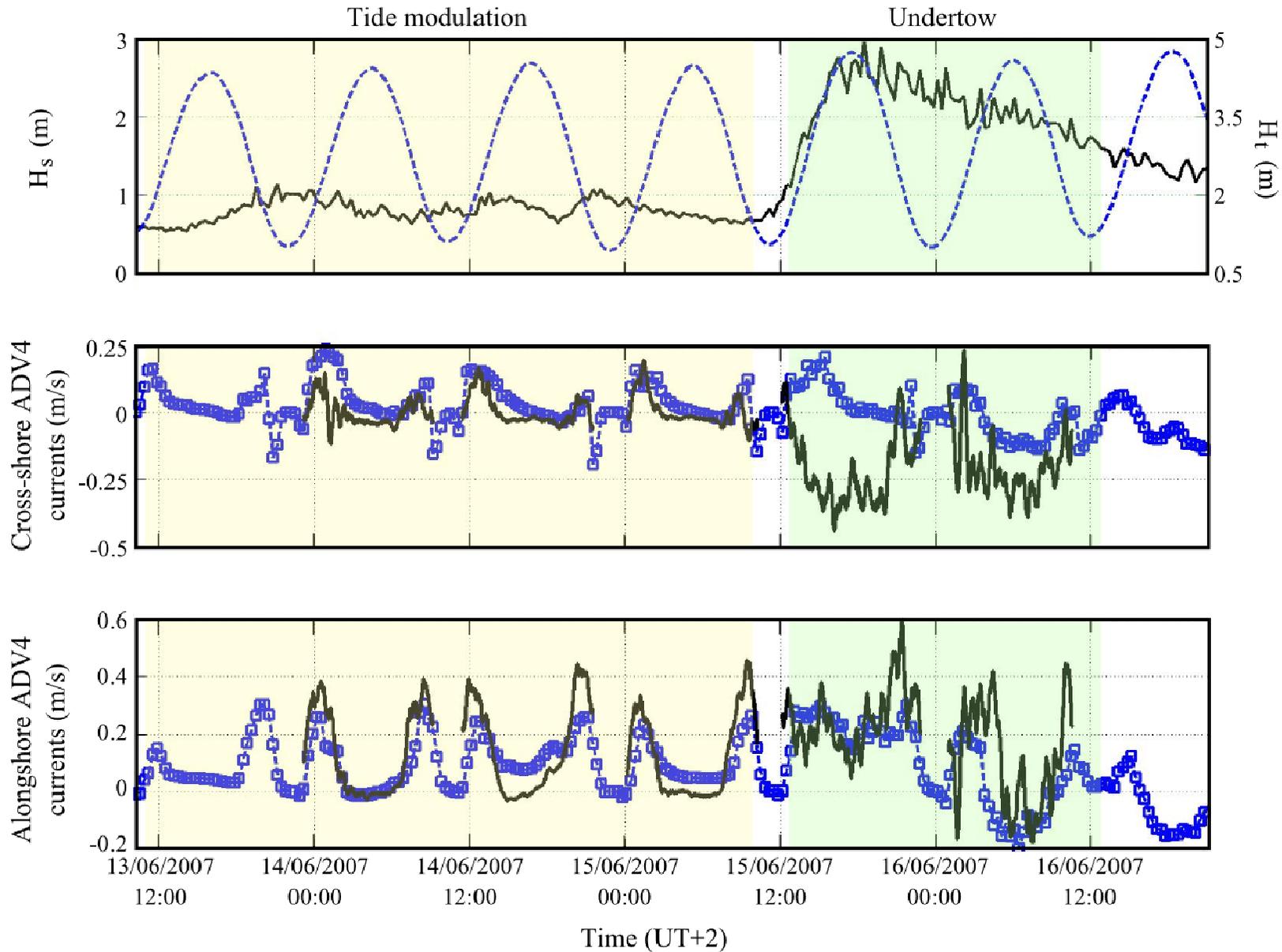


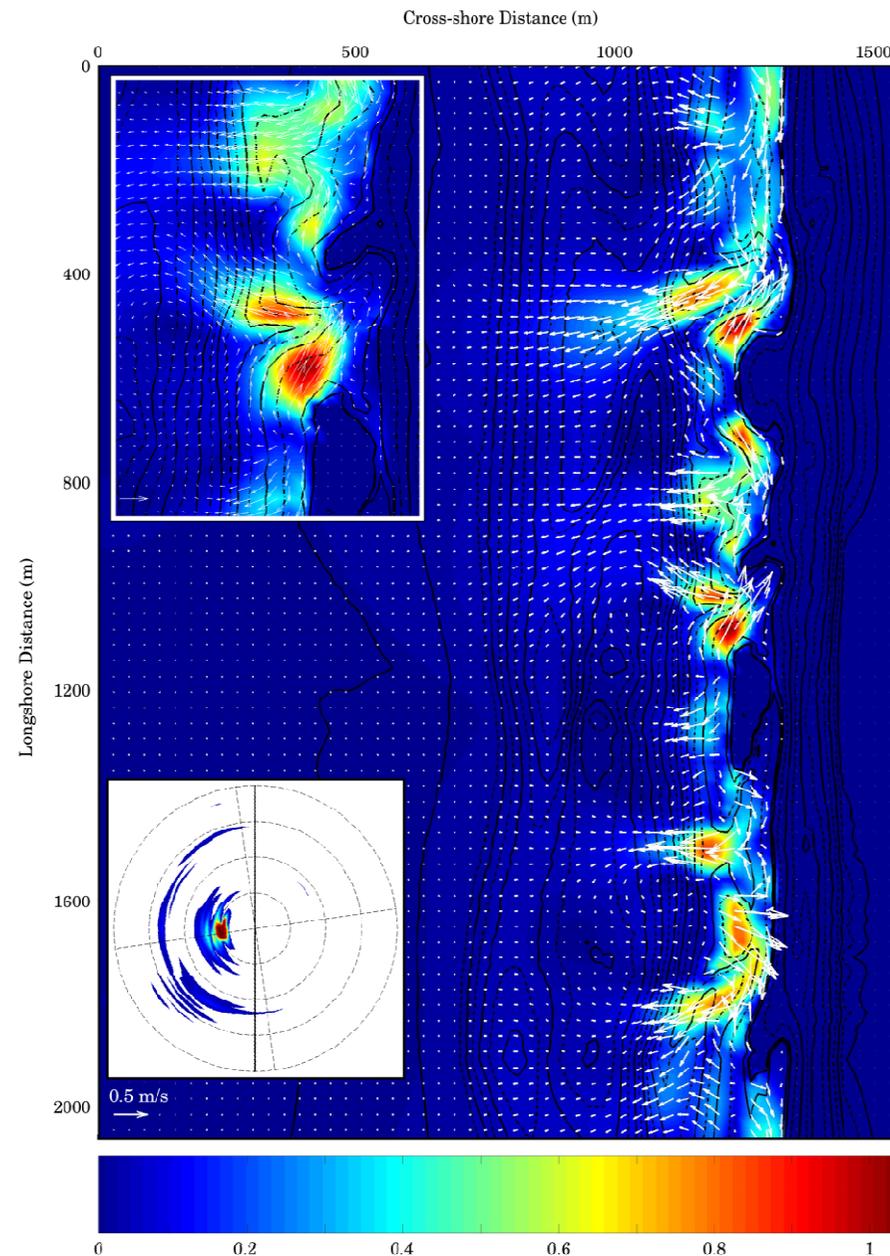




γ -parametrisation: Smith and Kraus, 1990

Intertidal zone measurements



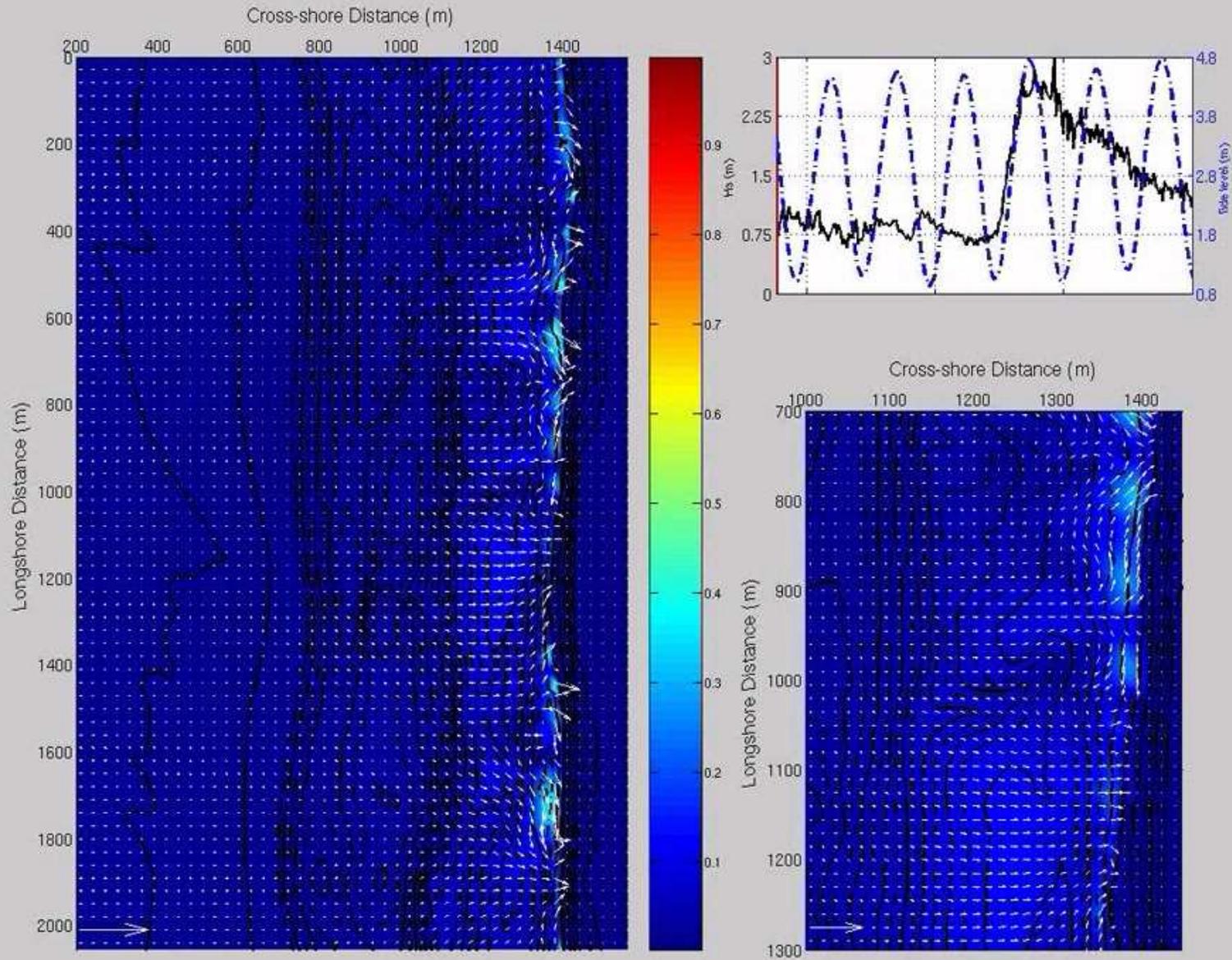


MARS / SWAN model

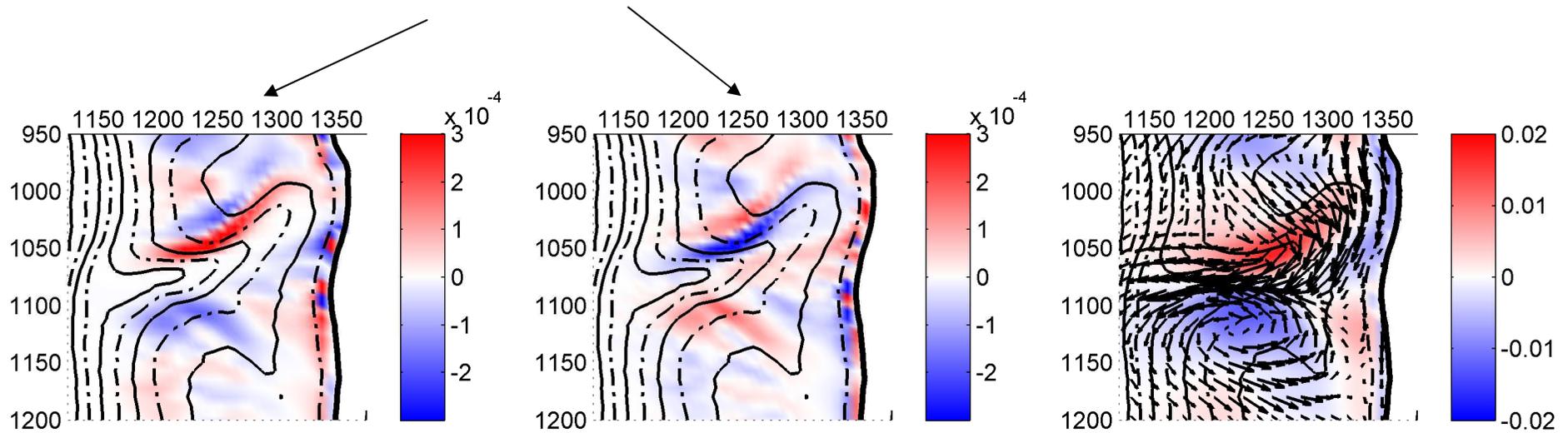
Bruneau, Bonneton, Castelle and Pedreros (2008)

Wave-induced circulation

Applications

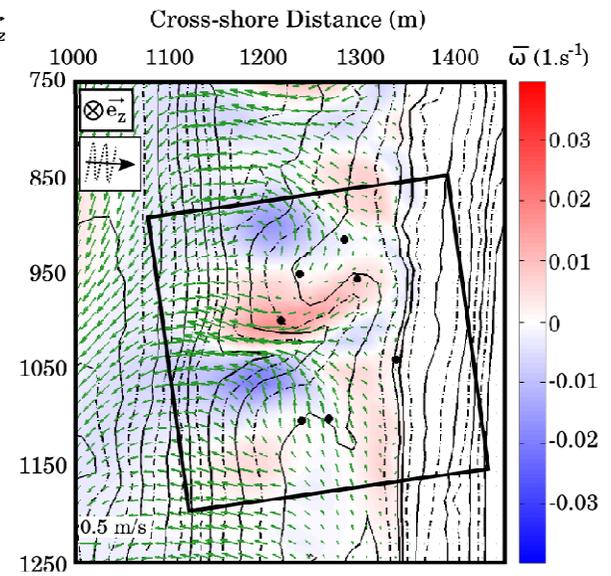
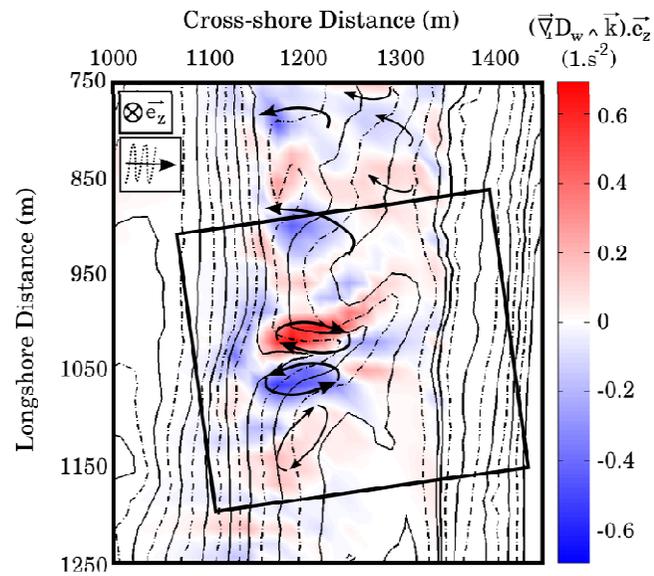
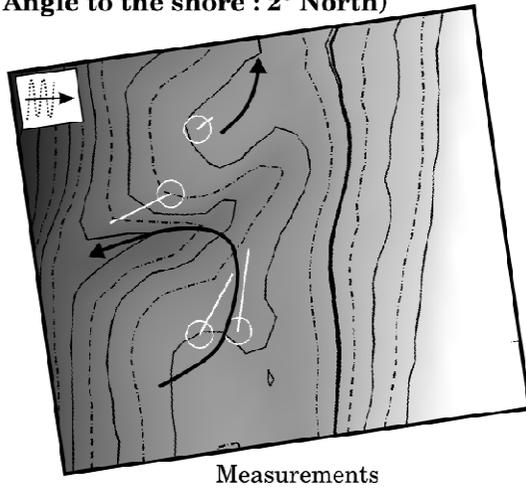


$$\nabla \mathcal{D} \wedge \mathbf{e}_k + \nabla \wedge \mathbf{V}_s \simeq 0$$



Hs=0.9m , $\theta = 10^\circ$, mid-tide

Energetic conditions
 (Hs=2.8m, Tp=10s,
 Angle to the shore : 2° North)



Bruneau, Bonneton, Castelle and Pedreros (2008)

Conclusion and perspectives

The rip current mean circulation is driven by wave-energy dissipation gradients, perpendicular to the direction of wave propagation, which are due to non-uniformities along the bore crests

$$\frac{\partial \omega}{\partial t} + \nabla \cdot \left(\omega \left(\mathbf{U} + \frac{\tilde{\mathbf{M}}}{\bar{h}} \right) \right) = \underbrace{\nabla \wedge (\mathcal{D} \mathbf{e}_k)}_{\sim \nabla \mathcal{D} \wedge \mathbf{e}_k} + \nabla \wedge \mathbf{V}_s + \nabla \wedge \mathbf{F}_r$$

$$\mathcal{D} = \frac{D_{bm}}{\bar{h} c_\phi}$$

vorticity (circulation) forcing term

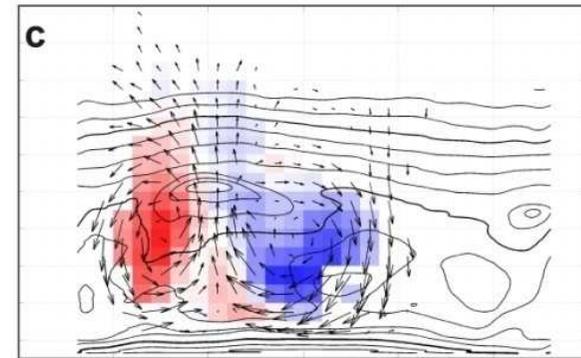
\mathcal{D} and v_t are the key parameters for describing rip current circulation

Conclusion and perspectives

Further works are required to better characterize ω , v_t and \mathcal{D}

□ $\omega, v_t \Rightarrow$ drifter-based method

- laboratory experiments: e.g. *Castelle et al. (2010)*
- field experiments: e.g. *MacMahan et al. (2009)*



□ $\mathcal{D} \Rightarrow$ theoretical NSW shock-wave approaches

$$\nabla \wedge (De_k)$$

$$\mathcal{D} = \frac{g}{4c_b T} \frac{(h_2 - h_1)^3}{h_2 h_1}$$

Bonneton et al. (2010)

\Rightarrow quantitative experimental validations

