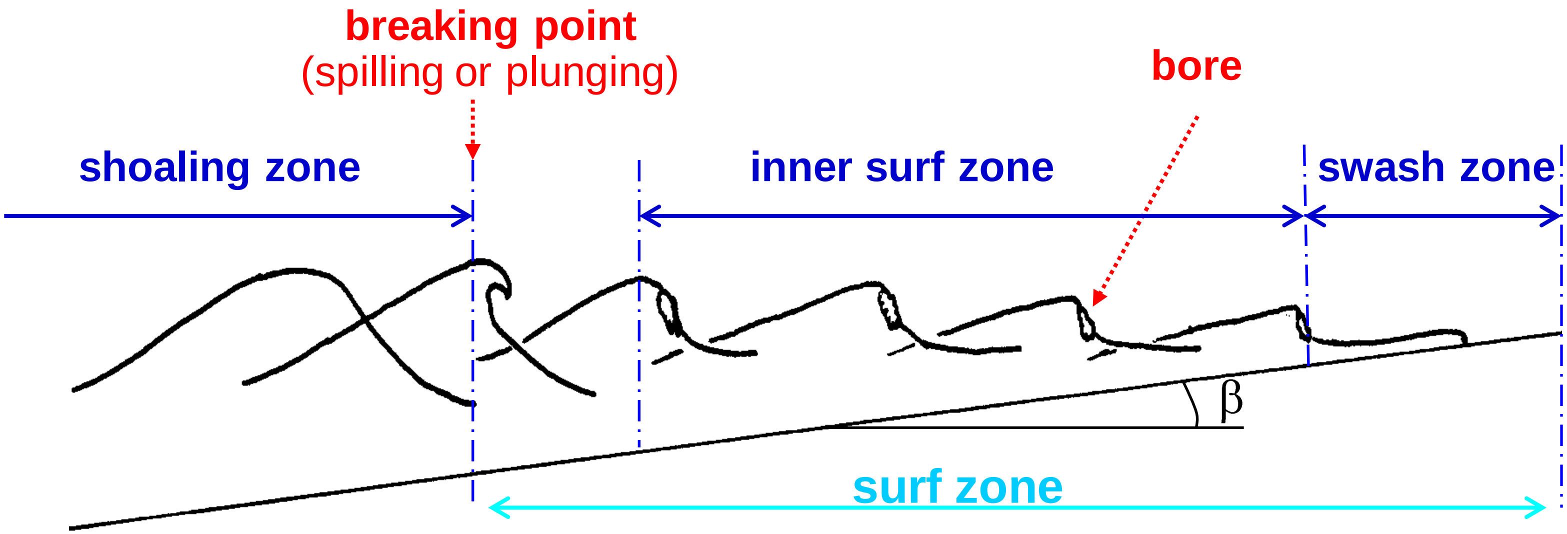
Conference "Oceanography and Mathematics", ENS, January 26-28

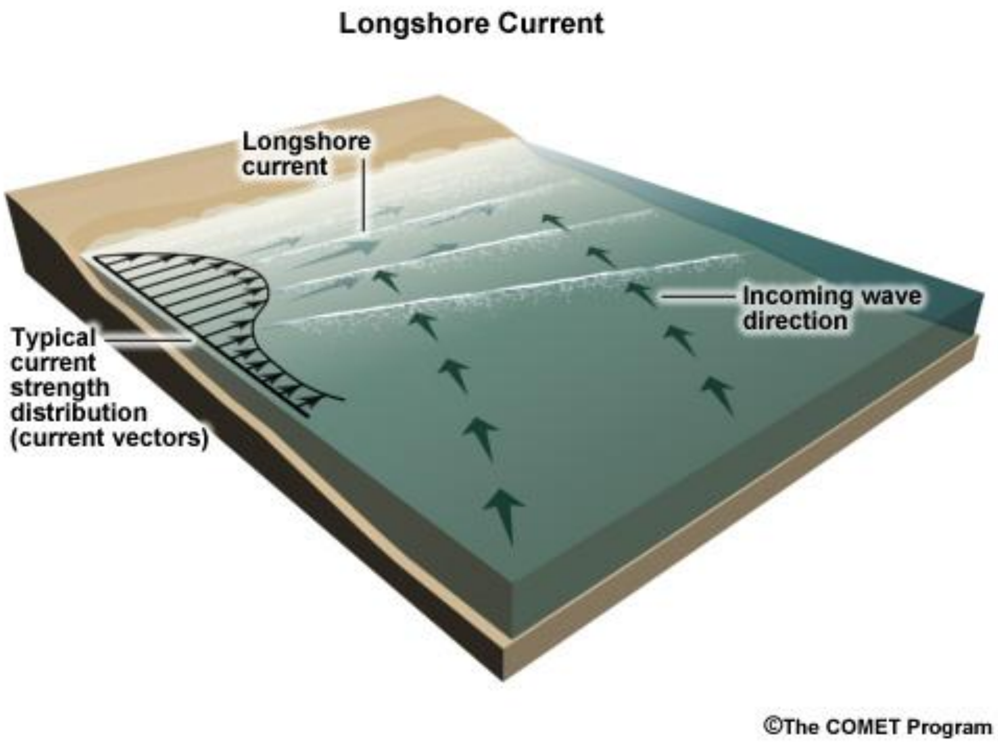
Modelling of wave-induced nearshore circulation

Philippe Bonneton

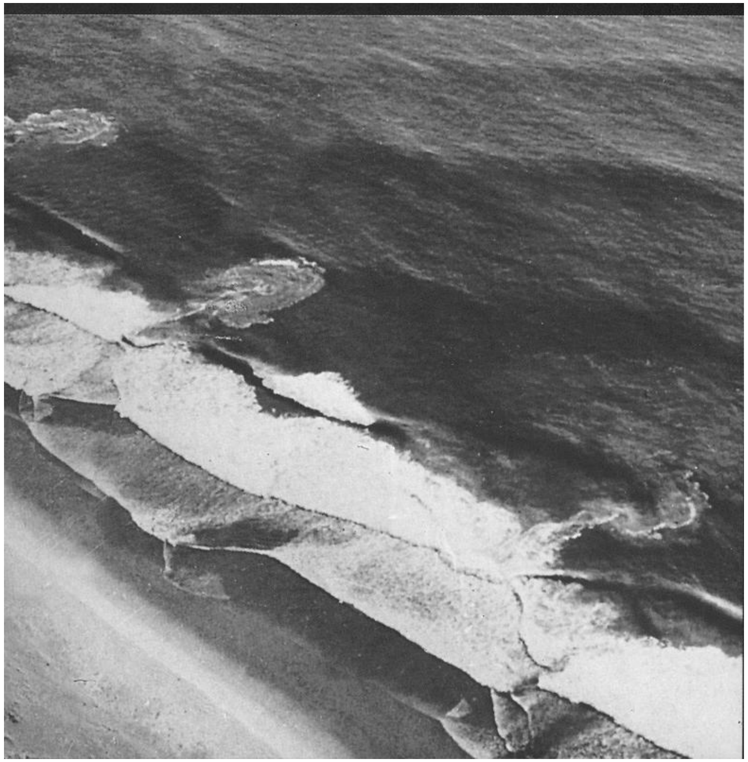
EPOC, Univ. Bordeaux I, CNRS


breaking point
(spilling or plunging)
bore
shoaling zone
inner surf zone
swash zone
β
surf zone


breaking point
bore

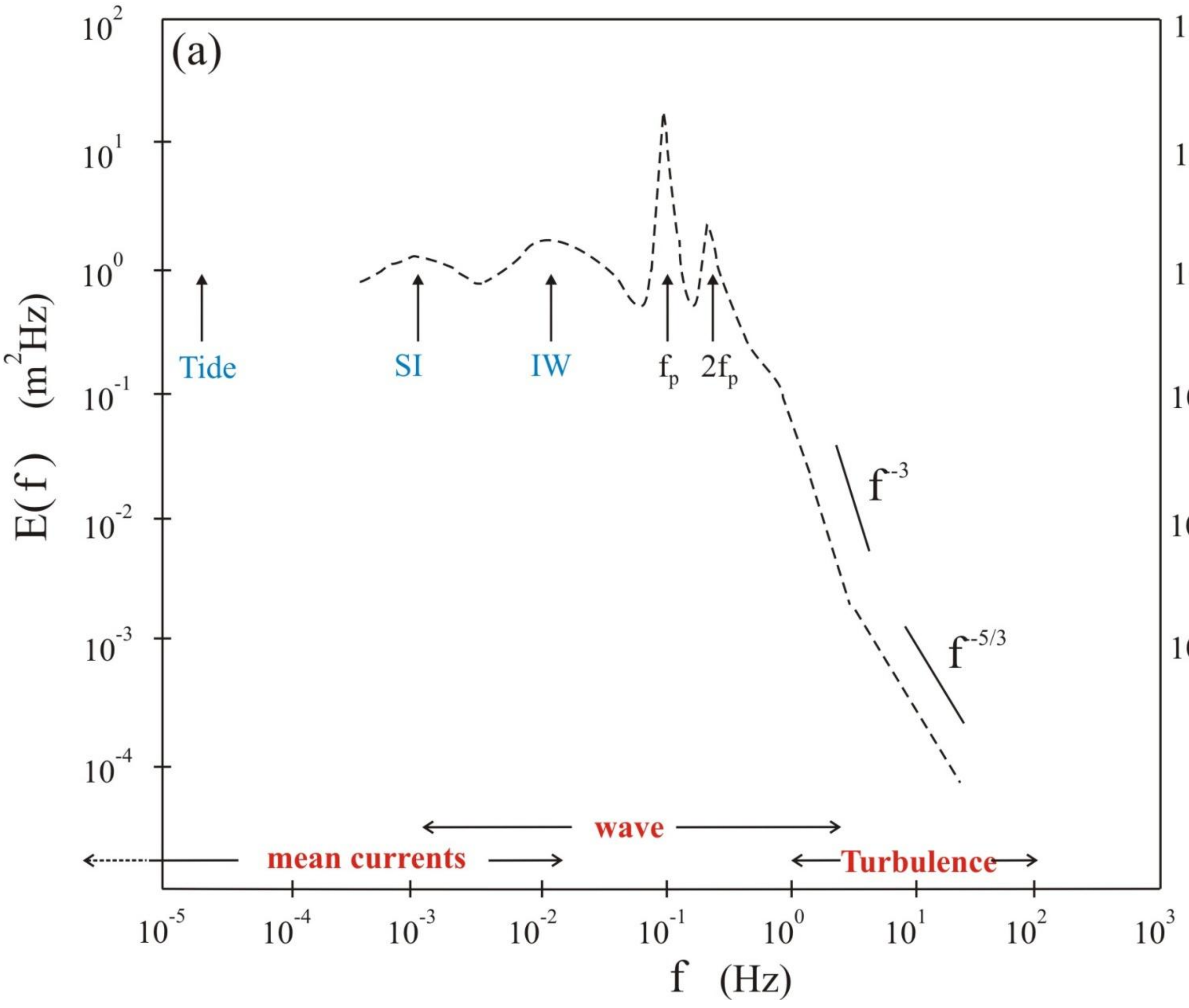


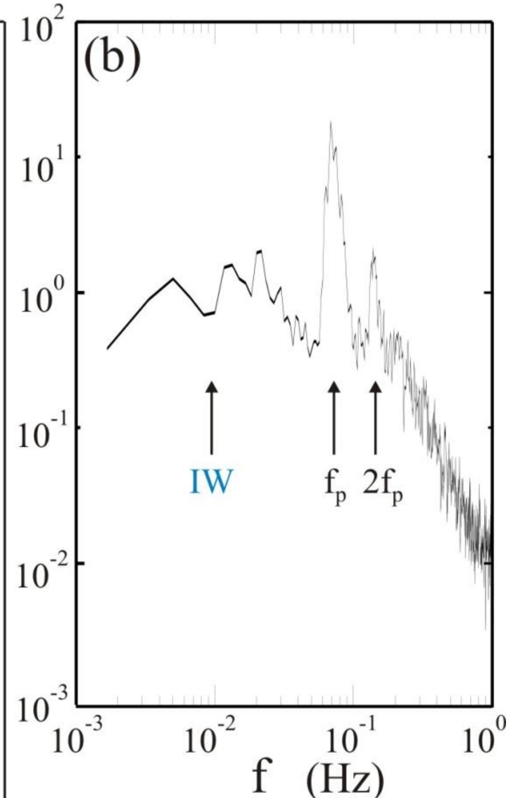
Longshore current



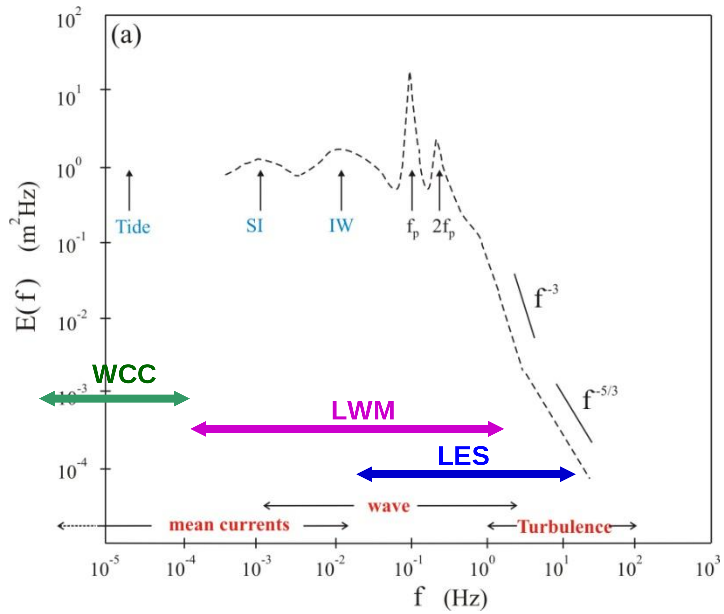
Rip currents and vortices

Energy spectrum of cross-shore velocity



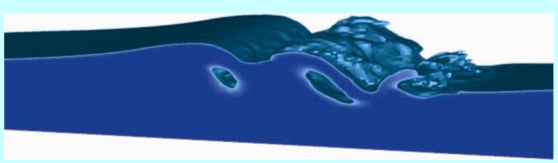


Bonneton (2003)



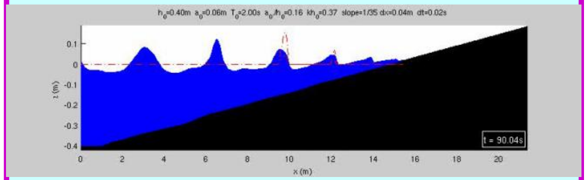
small scales — large scales

Two-phase flow LES



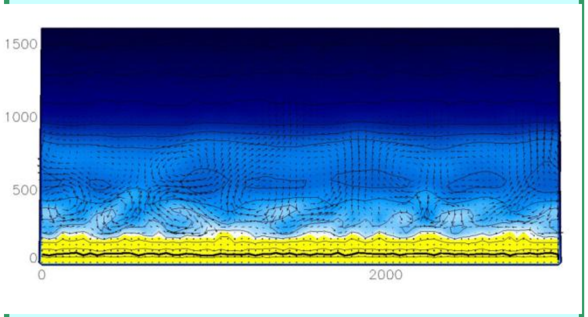
Lubin et al. (2006)

long wave modeling



Cienfuegos et al. (2006)

wave-current coupling



Castelle et al. (2006)

Long wave modeling:

Barthélémy, E. (LEGI, Grenoble), Cienfuegos, R. (PUC, Chile),
Lannes, D. (ENS, Paris), Marche, F. (I3M, Montpellier)

Wave-current coupling:

Bruneau, N. (EPOC, Bordeaux), Castelle, B. (EPOC, Bordeaux)
Pedreros, R. (BRGM, Orléans)

Research Programs:

- *Surf zone hydrodynamics* (IDAO/INSU, CNRS),
- *MODLIT* (RELIEFS/INSU, CNRS)
- *MathOcean* and *MISEEVA* (ANR)
- *ECORS* (SHOM)

Outline of the presentation

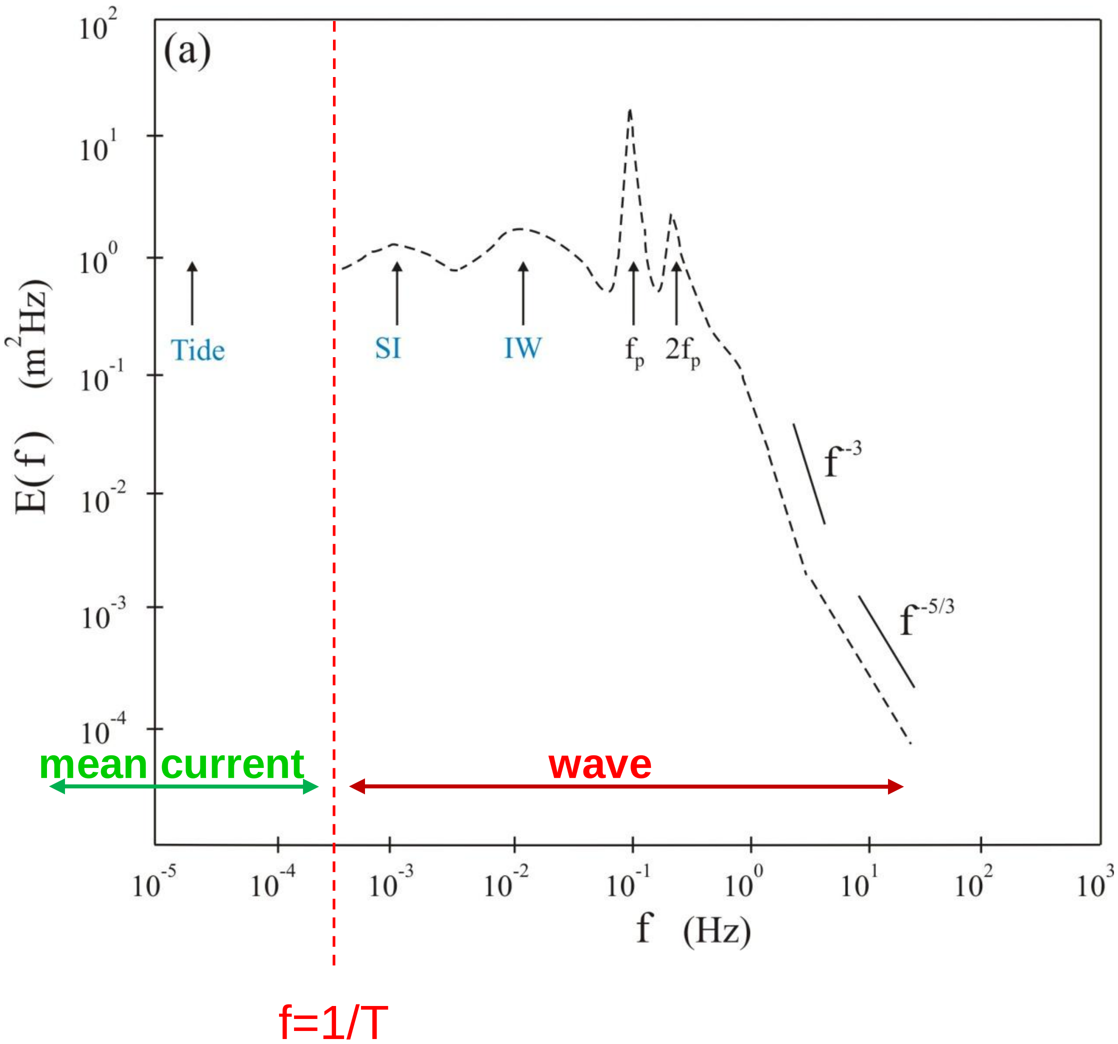
I – Introduction

II – Wave-induced currents and creation of vorticity due to dissipating (breaking) waves

III – Wave energy dissipation modeling

IV – Conclusions and perspectives

The flow is separated into **mean and wave components**:

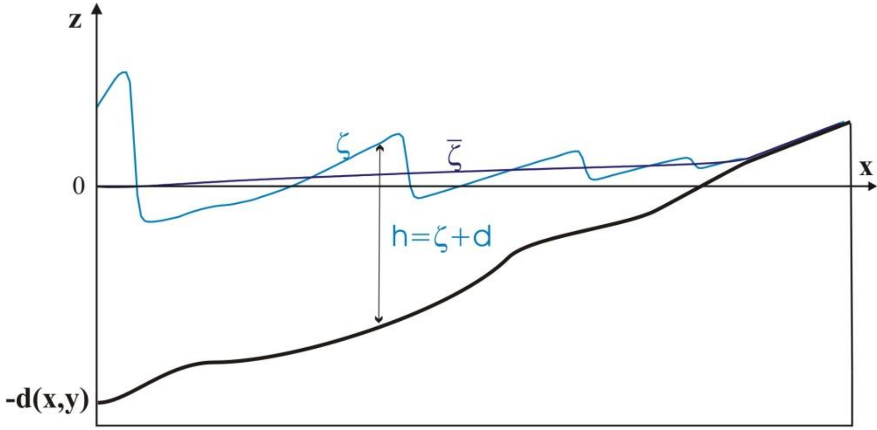


$$v_i = \bar{v}_i + \tilde{v}_i$$

$$\bar{v}_i = \frac{1}{T}\int_t^{t+T} v_i \, d\tau$$

Classical 2DH theory:

Longuet-Higgins (1964), Phillips (1977)



$$\frac{\partial \bar{h}}{\partial t} + \frac{\partial \bar{M}_j}{\partial x_j} = 0$$

$$\frac{\partial \bar{M}_i}{\partial t} + \frac{\partial}{\partial x_j}\left((\bar{M}_i \bar{M}_j)/\bar{h}\right) + g\bar{h}\frac{\partial \bar{\zeta}}{\partial x_i} = -\frac{\partial S_{ij}}{\partial x_j}$$

$$\bar{M}_i = \overline{\int_{-d}^{\zeta} v_i \, dz} = \bar{h}U_i + \tilde{M}_i$$

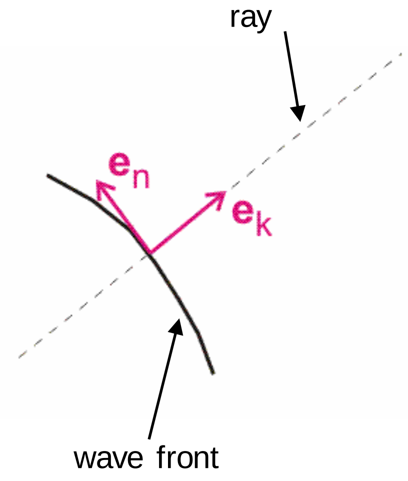
radiation stress: $S_{ij} = \overline{\int_{-d}^{\zeta} (\mathcal{P}\delta_{ij} + \rho \tilde{u}_i \tilde{u}_j) \, dz} - \frac{1}{2}\rho g \bar{h}^2 \delta_{ij} - \rho \frac{\tilde{M}_i \tilde{M}_j}{\bar{h}}$

Smith (2006)

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_j}\left(A(c_{gj} + U_j)\right) = -\frac{D_{b_m}}{\sigma}$$

$$A = E/\sigma \qquad \tilde{M}_i = Ak_i$$

$$\begin{aligned}\frac{\partial \bar{h}}{\partial t} + \nabla.(\bar{h}\mathbf{U}) &= -\nabla.\tilde{\mathbf{M}} \\ \frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}.\nabla)\mathbf{U} + g\nabla\bar{\zeta} &= \mathcal{D}\mathbf{e}_k + \frac{\tilde{\mathbf{M}}}{\bar{h}} \wedge (\nabla \wedge \mathbf{U}) - \nabla\tilde{J}\end{aligned}$$

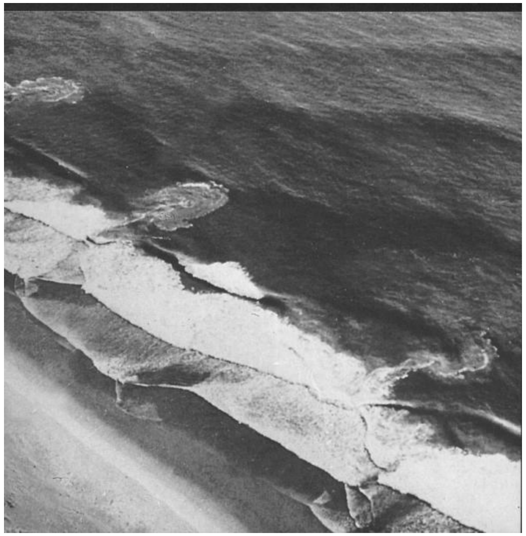


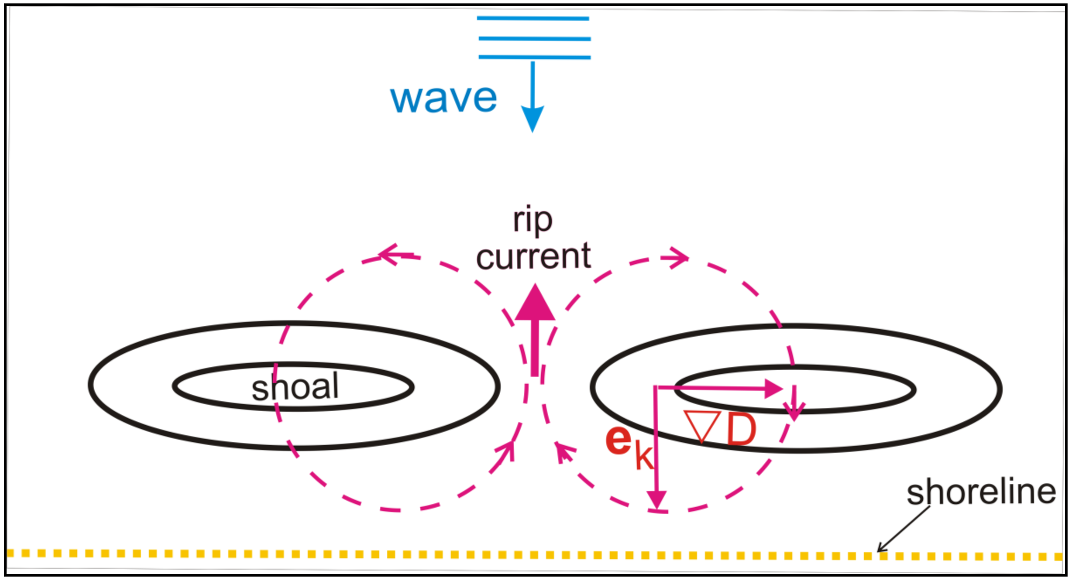
$$\tilde{J} = E\frac{k}{\sinh(2k\bar{h})} \qquad \mathcal{D} = \frac{D_{b_m}}{\bar{h}c_\phi}$$

Vorticity equation

$$\frac{\partial \omega^m}{\partial t} + \nabla . \left(\omega^m (\mathbf{U} + \frac{\tilde{\mathbf{M}}}{\bar{h}}) \right) = \nabla \mathcal{D} \wedge (\mathbf{e}_k)$$

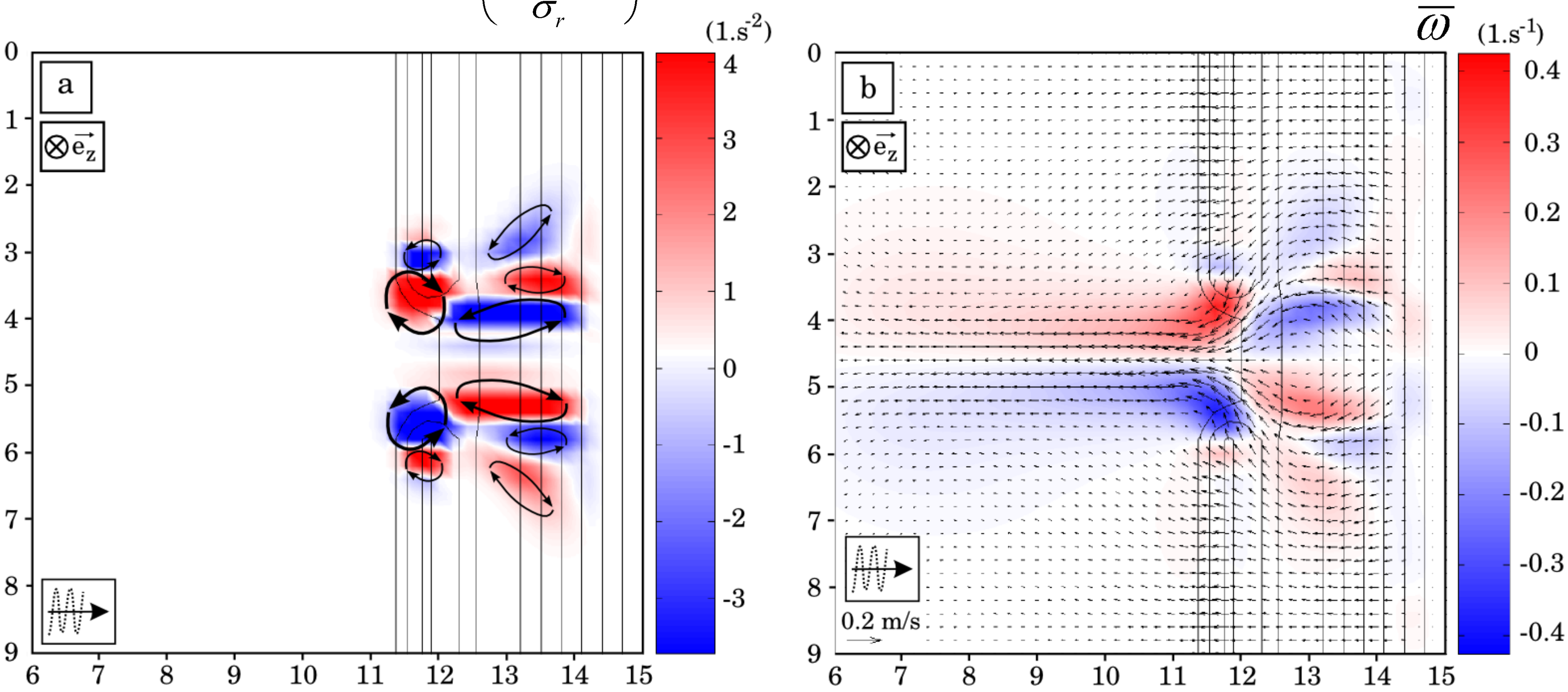
(see also Bühler, 2000)



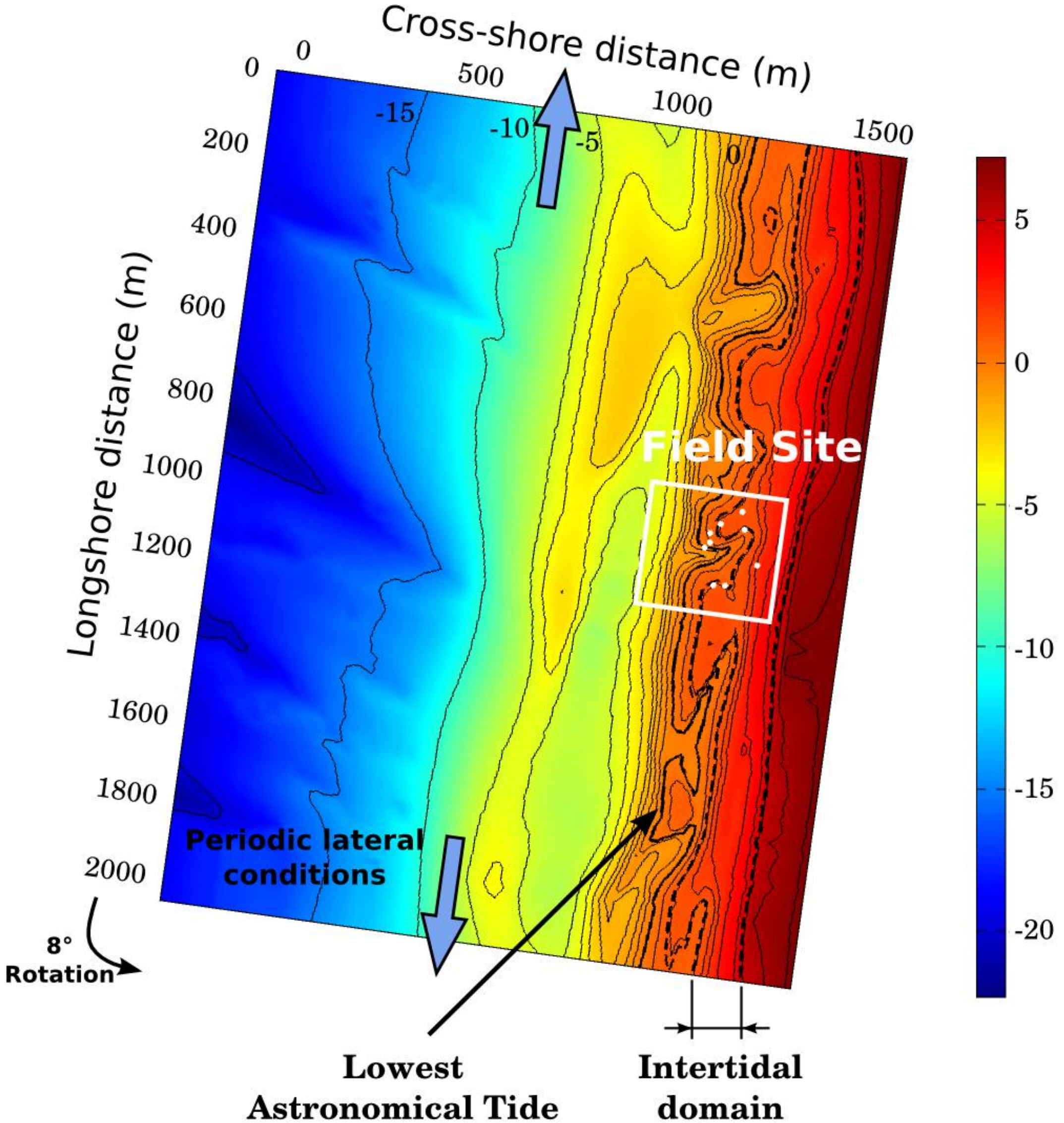


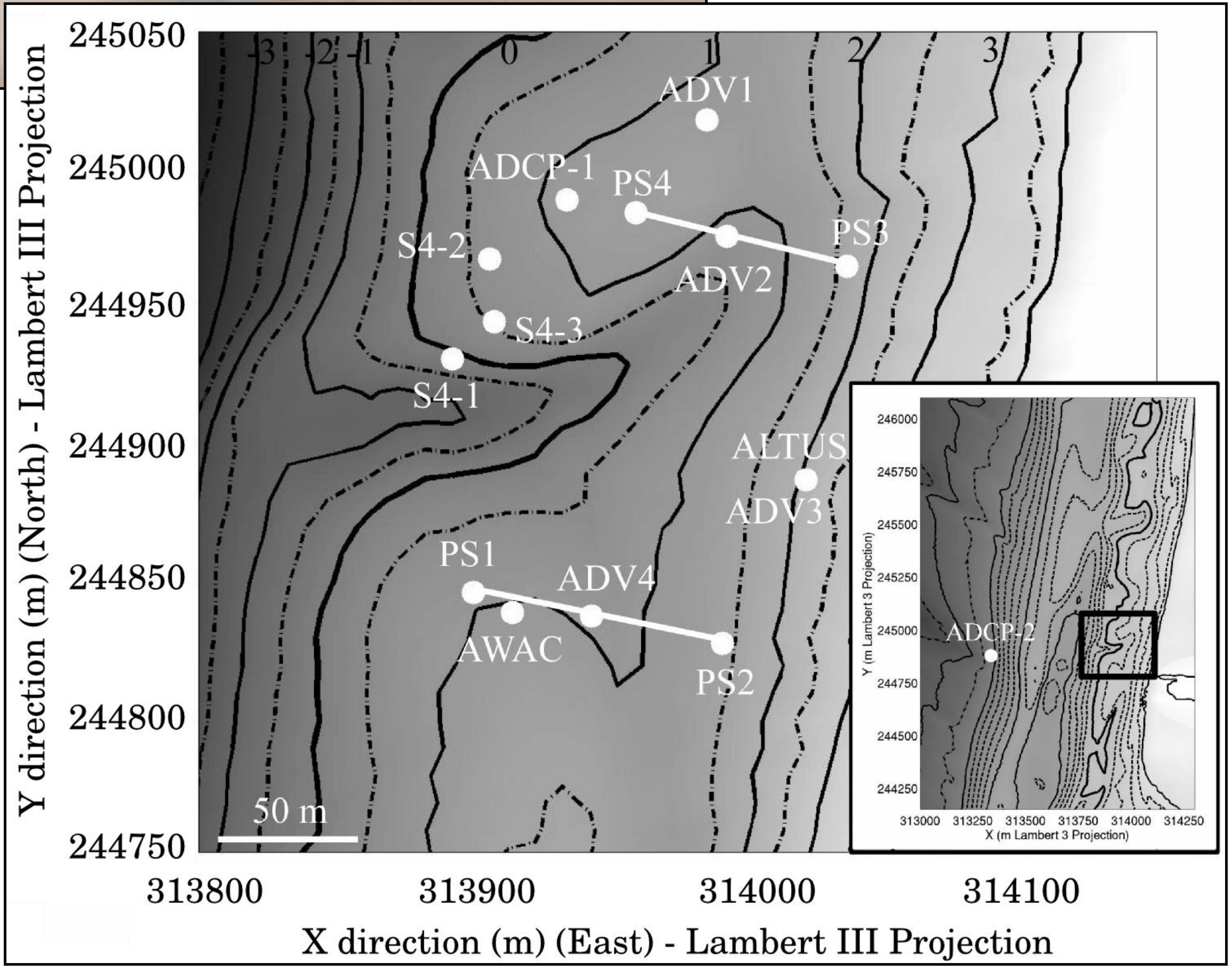
Bruneau N. (PhD 2008) ⇨ coupling of MARS 2D (IFREMER) and SWAN (TU Delft)

$$\left(\vec{\nabla}\frac{D_r}{\sigma_r}\wedge\vec{k}\right).\vec{e}_z$$



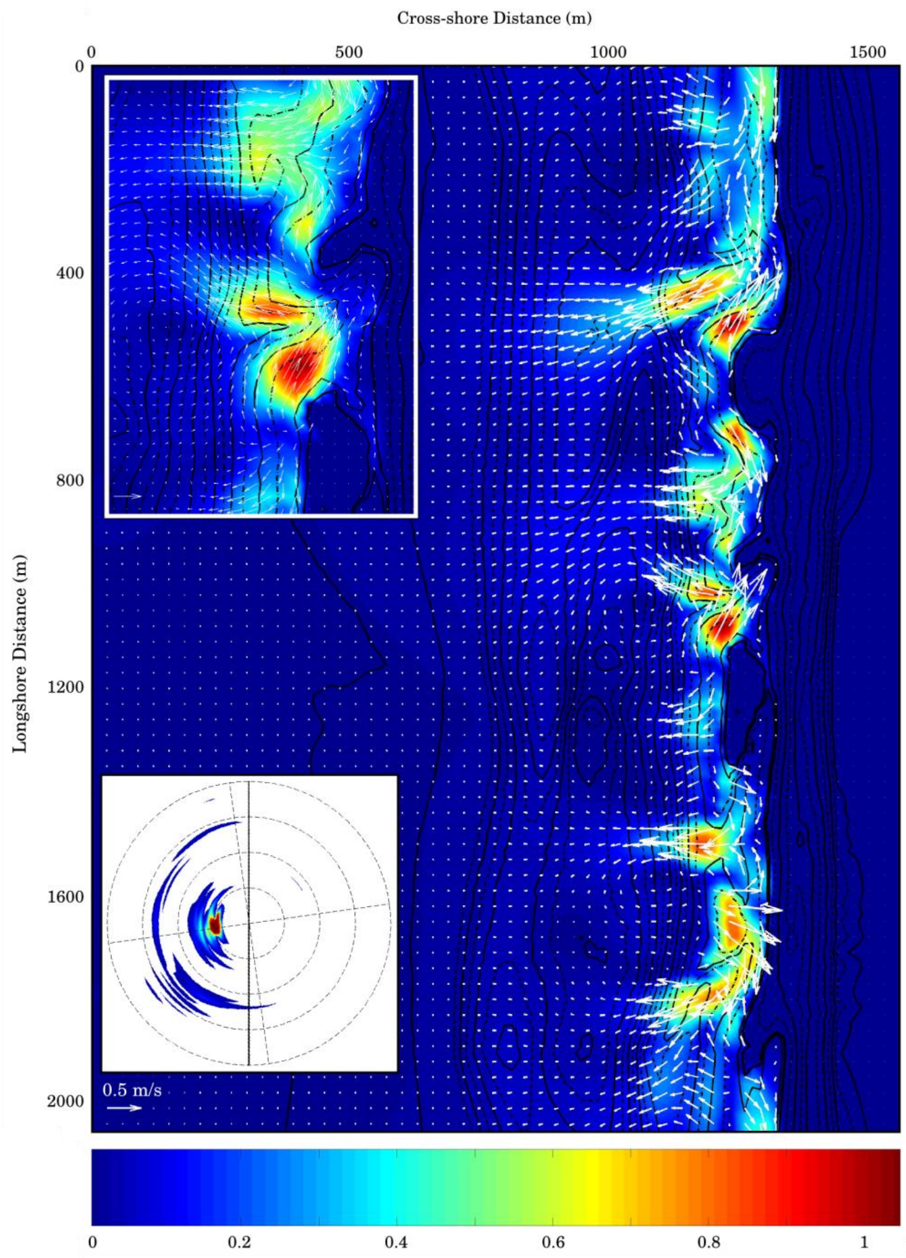
Biscarosse Beach 2007

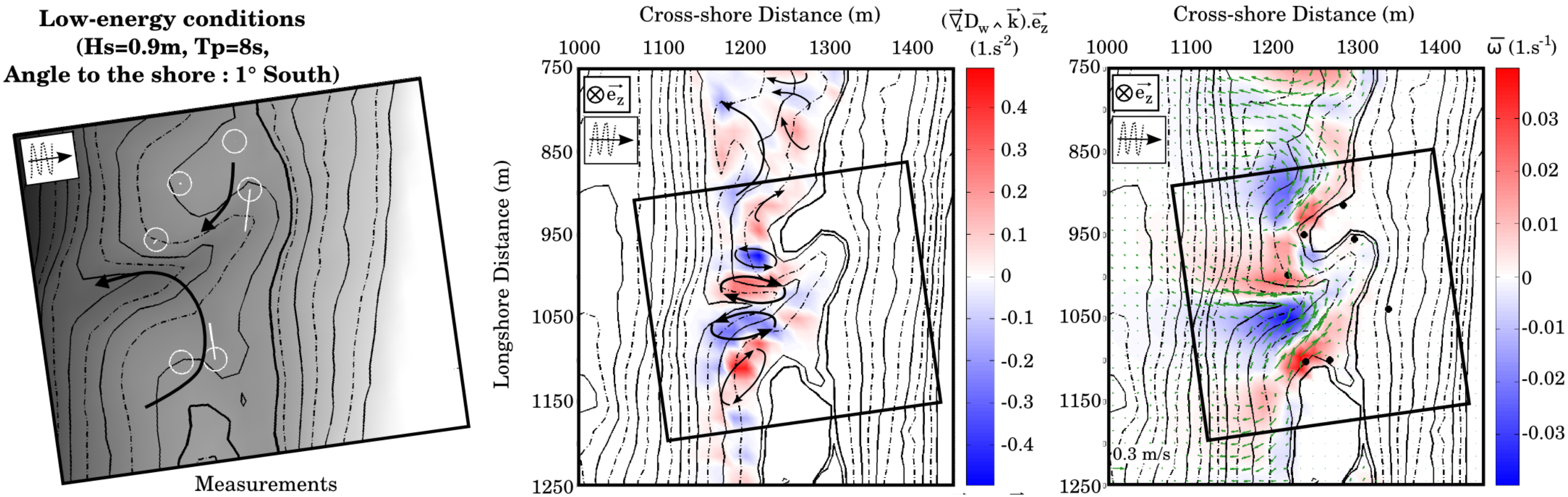




Y direction (m) (North) - Lambert III Projection
245050
245000
244950
244900
244850
244800
244750
313800
313900
314000
314100
X direction (m) (East) - Lambert III Projection
-3
-2
-1
0
1
2
3
ADV1
ADCP-1
PS4
PS3
S4-2
ADV2
S4-3
S4-1
ALTUS
ADV3
PS1
ADV4
AWAC
PS2
50 m
ADCP-2
Y (m Lambert 3 Projection)
X (m Lambert 3 Projection)

II – wave-induced currents and vorticity | applications





Bruneau, Bonneton, Castelle and Pedreros (2008)

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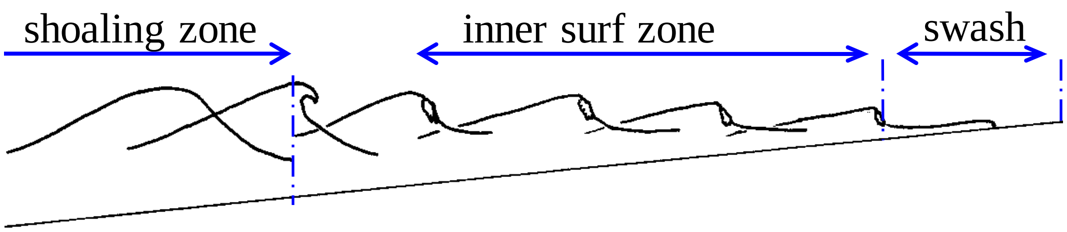
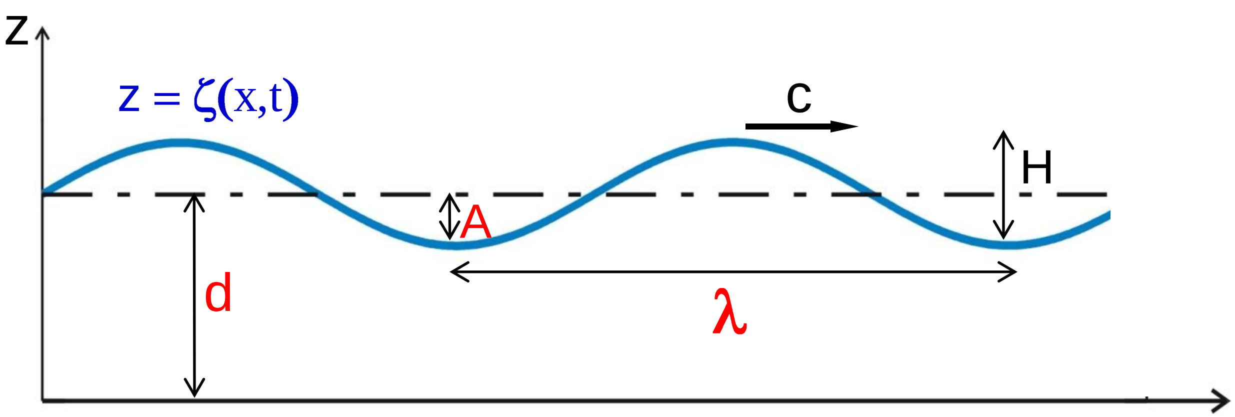
Nearshore zone





$$\varepsilon = \frac{A}{d}$$

$$\mu = \left(\frac{d}{\lambda}\right)^2$$

$$\mu \leq 0.01$$

Green Naghdi (1976) equations

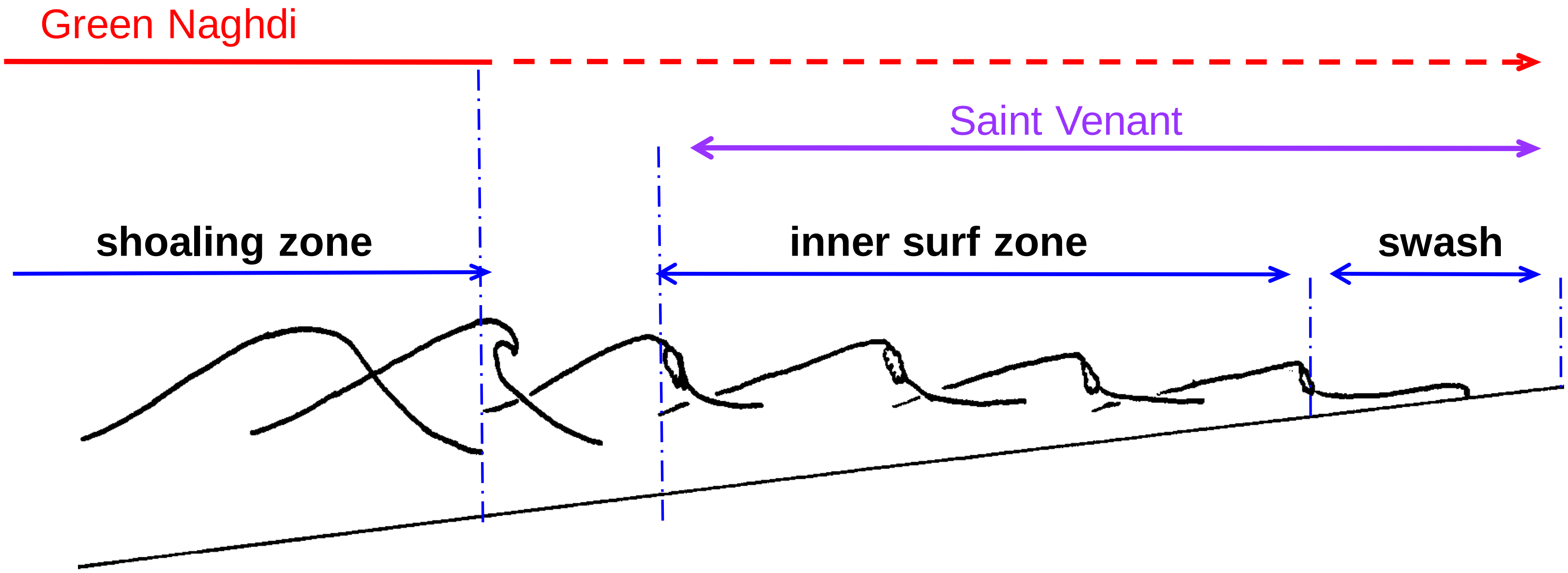
$$\begin{aligned} \partial_t \zeta + \nabla \cdot (h\mathbf{v}) &= 0 \\ \partial_t \mathbf{v} + \varepsilon(\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla\zeta &= \mu\frac{\mathbf{S}}{h} \end{aligned} \quad + \ O(\mu^2)$$

$$\begin{aligned} \mathbf{S} &= \frac{\epsilon}{3}\nabla[h^3((\mathbf{v}\cdot\nabla)(\nabla\cdot\mathbf{v}) - (\nabla\cdot\mathbf{v})^2)] \\ &+ \frac{1}{3}\nabla(h^3\nabla\cdot\partial_t\mathbf{v}) + \frac{1}{2}[\nabla(h^2\nabla d\cdot\partial_t\mathbf{v}) + h^2\nabla d\nabla\cdot\partial_t\mathbf{v}] - h\nabla d\nabla d\cdot\partial_t\mathbf{v} \\ &+ \frac{\epsilon}{2}[\nabla(h^2(\mathbf{v}\cdot\nabla)^2 d) + h^2((\mathbf{v}\cdot\nabla)(\nabla\cdot\mathbf{v}) - (\nabla\cdot\mathbf{v})^2)\nabla d] - h((\mathbf{v}\cdot\nabla)^2 d)\nabla d \end{aligned}$$

Green Naghdi equations represent the appropriate model to describe nonlinear shallow water wave propagation in the nearshore and wave oscillations at the shoreline.

(see *Lannes et Bonneton (Phys. Fluids, 2009)*)

How can we model the wave energy dissipation at wave fronts ?



Cienfuegos, Barthélémy et Bonneton. (2005, 2009)

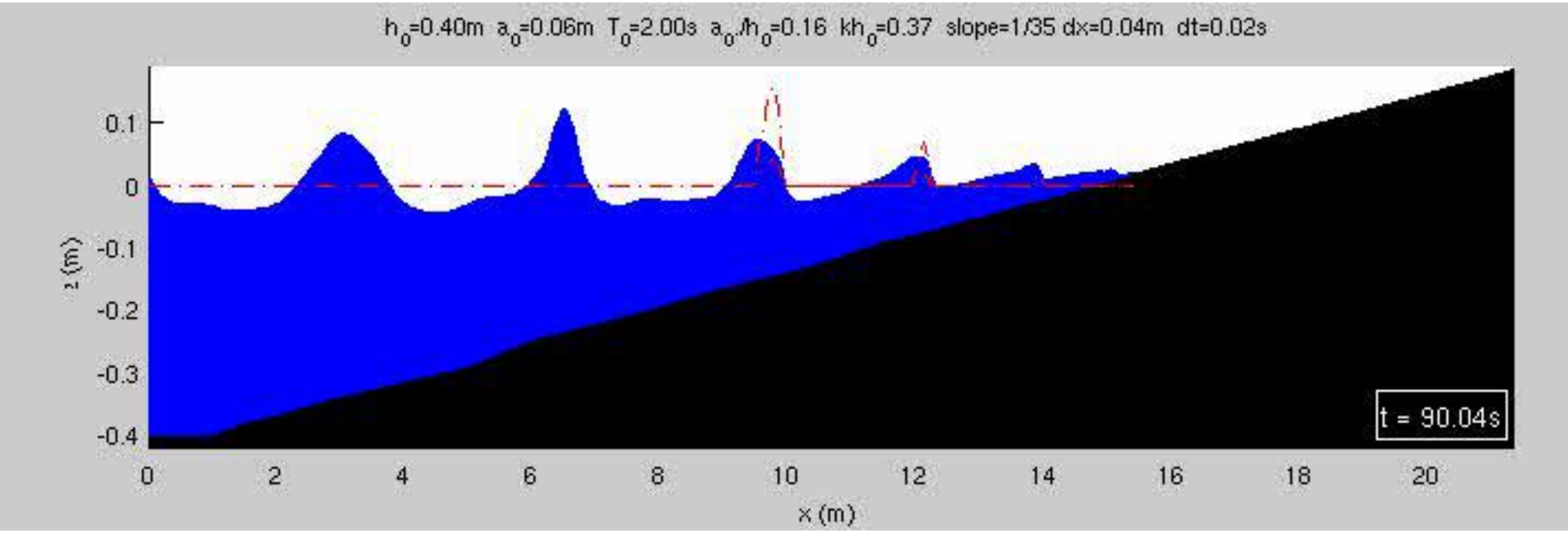
$$\begin{aligned} \partial_t \zeta + \nabla \cdot (h\mathbf{v}) &= D_h \\ \partial_t \mathbf{v} + \varepsilon(\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla\zeta &= \mu\frac{\mathbf{S}}{h} + \frac{1}{h}D_{hu} \end{aligned}$$

$$D_h = \frac{\partial}{\partial x}\left(\nu_h \frac{\partial h}{\partial x}\right) \qquad D_{hu} = \frac{\partial}{\partial x}\left(\nu_{hu} \frac{\partial (hu)}{\partial x}\right)$$

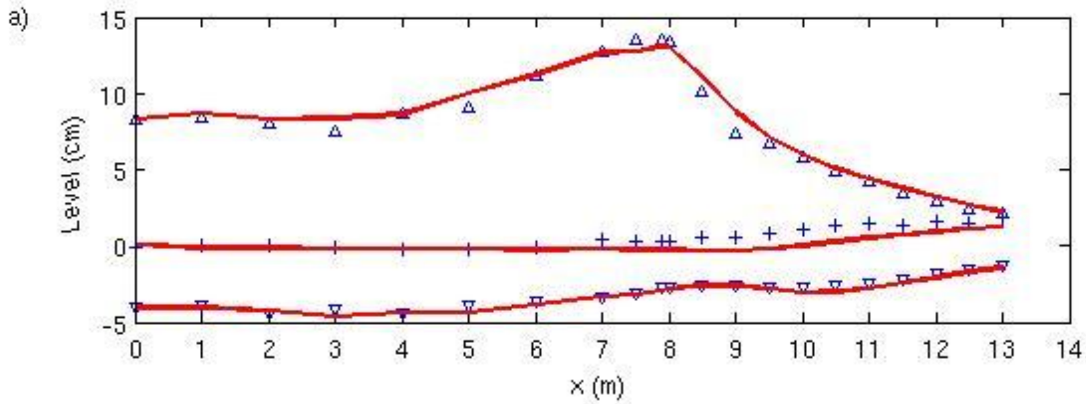
- Classical parametrization (Kennedy et al (2000)): $D_h = 0$

- Dutykh and Dias (2007):
  quasi-potential flow with a weak vortical component ⇨ D_h

Cienfuegos, Barthélémy et Bonneton. (2005, 2009)



Validation with Ting and Kirby (1994) data



Inner surf and swash zones ⇨ Saint venant equations

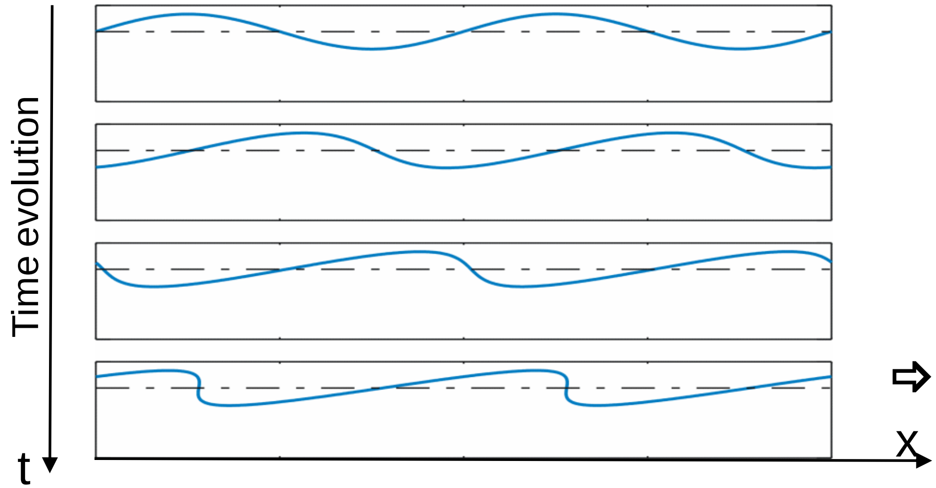
$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} = 0$$

$$\frac{\partial hu}{\partial t} + \frac{\partial}{\partial x}\left(hu^2 + \frac{1}{2}gh^2\right) = gh\frac{\partial d}{\partial x} - \frac{1}{2}f_r|u|u$$

SV ⟶ Euler

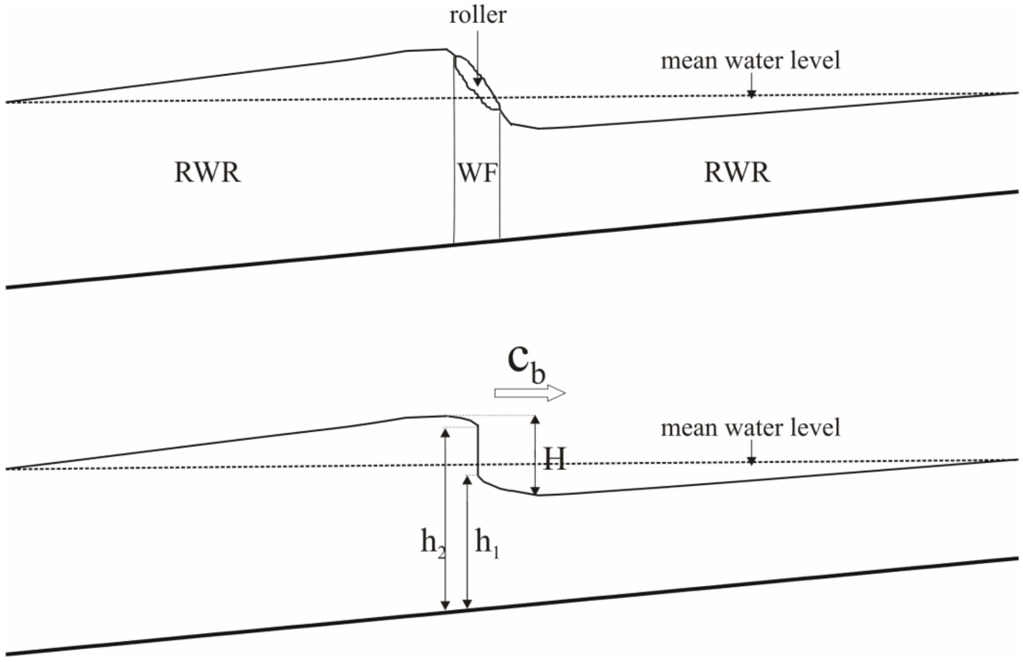
h ⟶ ρ

$F=u/\sqrt{gh}$ ⟶ $M_a=u/c_s$



⇨ **shock wave formation**

Wave front and shock wave



shock conditions:

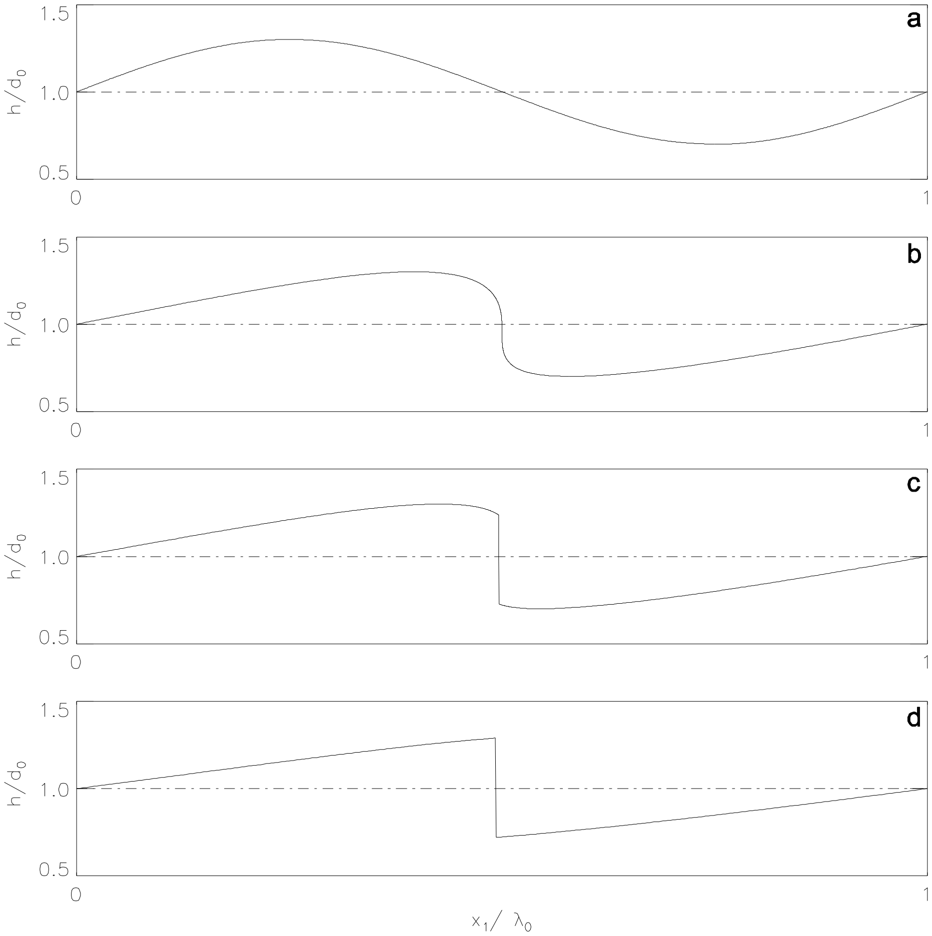
$$
\begin{aligned}
-c_b[h] + [hu] &= 0 \\
-c_b[hu] + [hu^2 + \frac{1}{2}gh^2] &= 0
\end{aligned}
$$

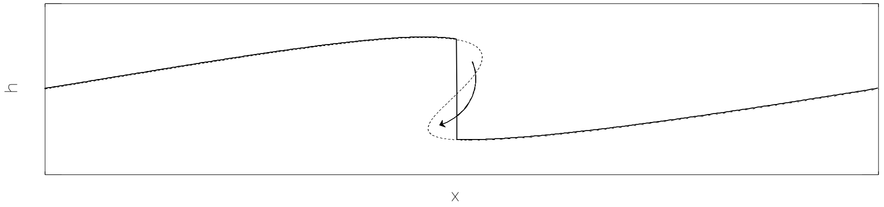
energy dissipation:

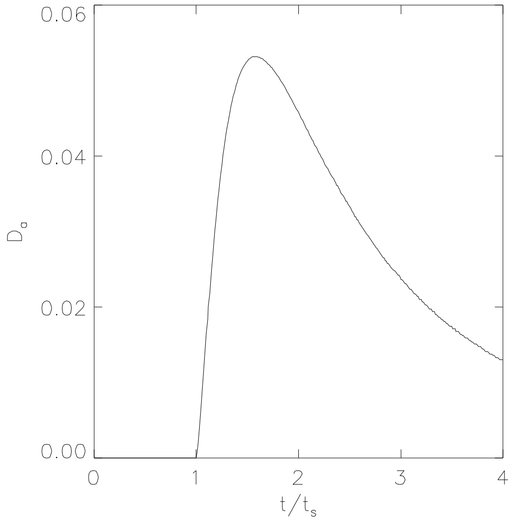
$$D_b = \frac{g}{4}\left(\frac{g(h_2+h_1)}{2h_1h_2}\right)^{1/2} (h_2 - h_1)^3$$

Do weak solutions of the Saint Venant equations can reproduce non-linear wave transformation and energy dissipation in the surf zone ?

One-way wave propagation on a flat bottom







2D shock-capturing finite-volume codes

- **SURF_SV**

  Mac-Cormack TVD scheme, 2^{nd} order, gently sloping beach

  Vincent, Caltagirone, Bonneton (2001)

- **SURF_WB**

  Positivity preserving high order VFRoe solver,
  well-balance « hydrostatic Reconstruction Method » (*Audusse et al. (2005)*)

  Marche, Bonneton, Fabrie, Seguin (2007), Marche and Berthon (2008)

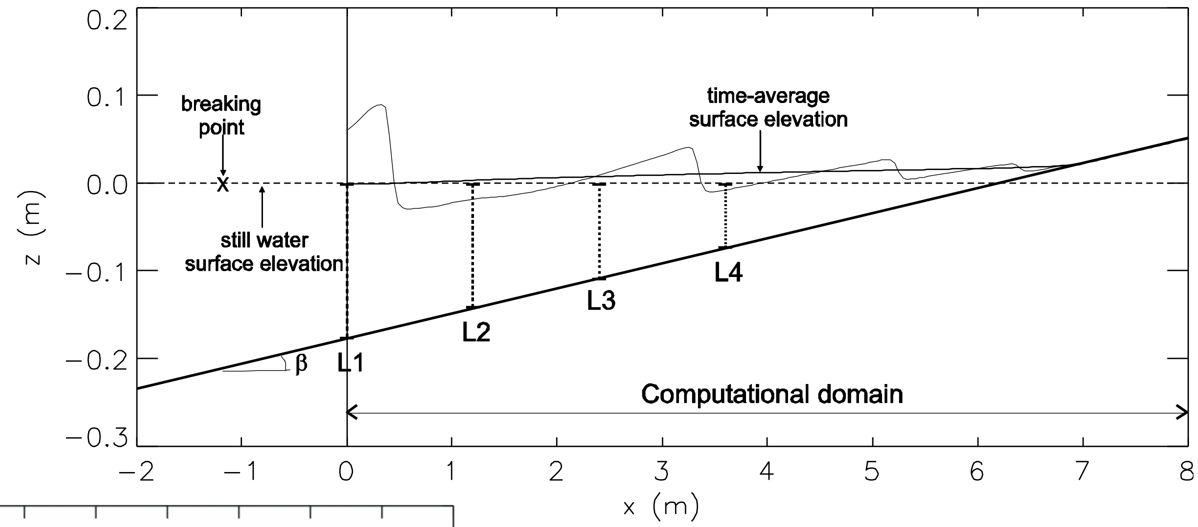
Comparison between shock-capturing numerical simulations (SURF_SV) and laboratory experiments

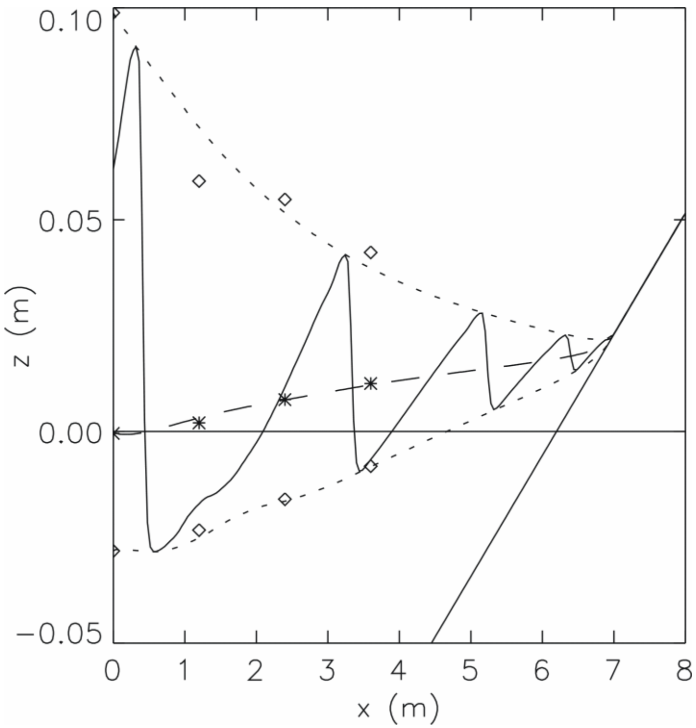
Cox's (1995) spilling breaking experiment_

H_0=0.115 m, T=2.2 s, β=1/35

Numerical parameters

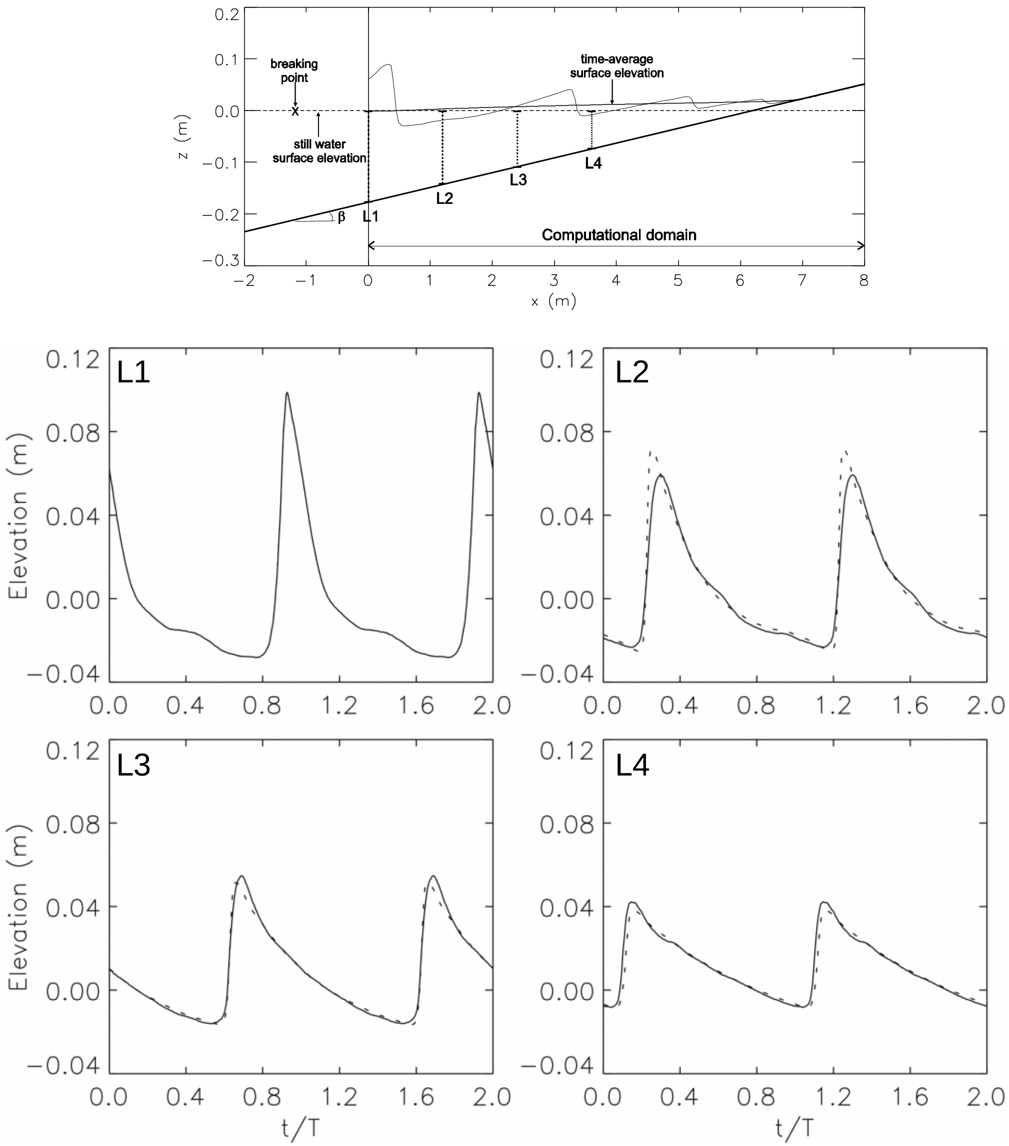
Δx=0.04 m, Δt=0.01 s, f_r=0.015





Bonneton (2007)

Comparison between shock-capturing numerical simulations (SURF_SV) and laboratory experiments

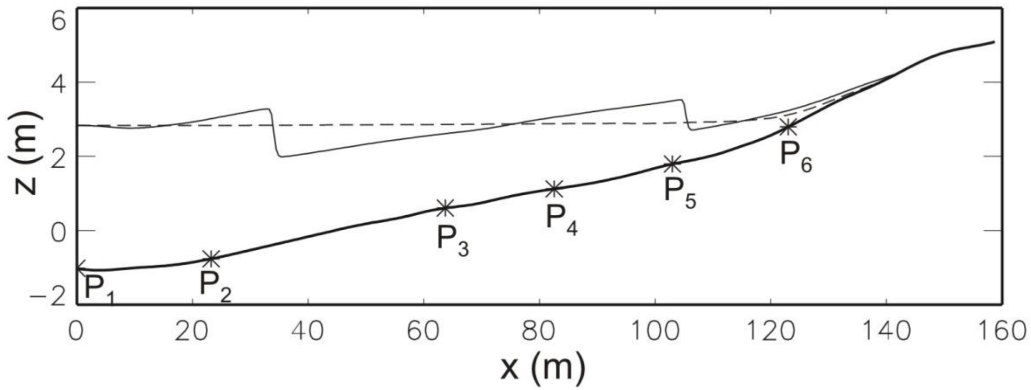


Comparison with field data

Bonneton et al (2004)

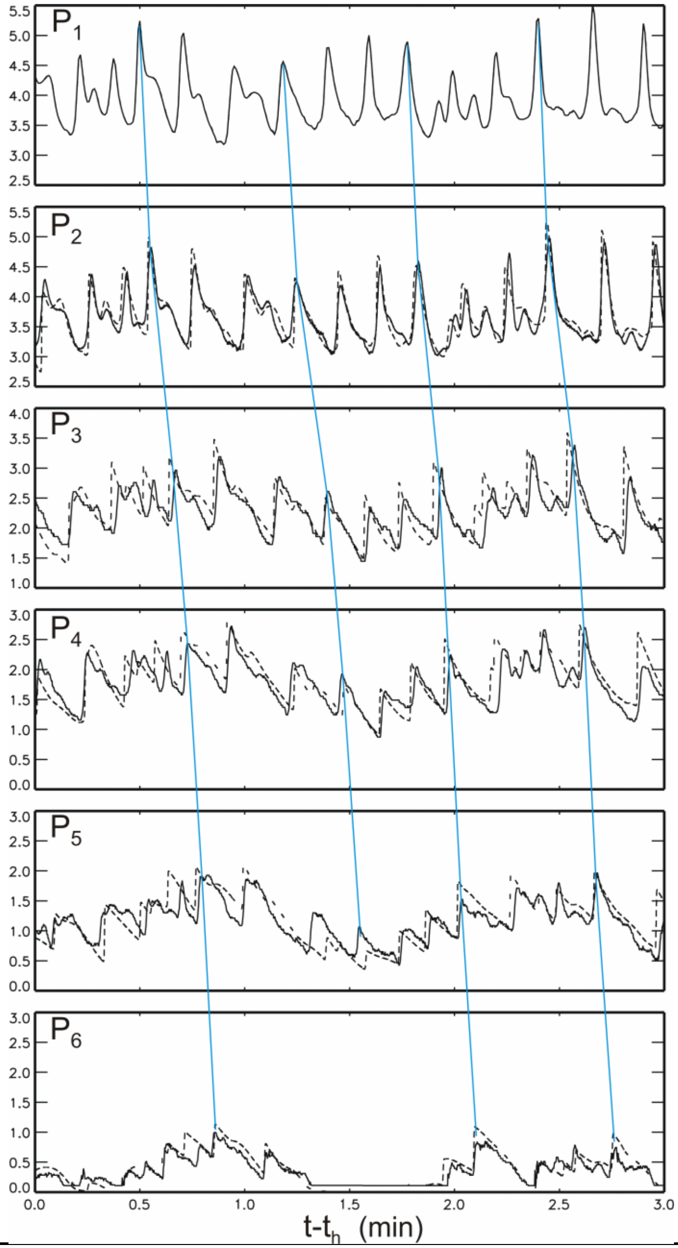
Truc Vert Beach 2001

- Offshore wave conditions: $\theta \approx 0°$, Hs=3 m, Ts=12 s
- Maximum surf zone width: 500 m

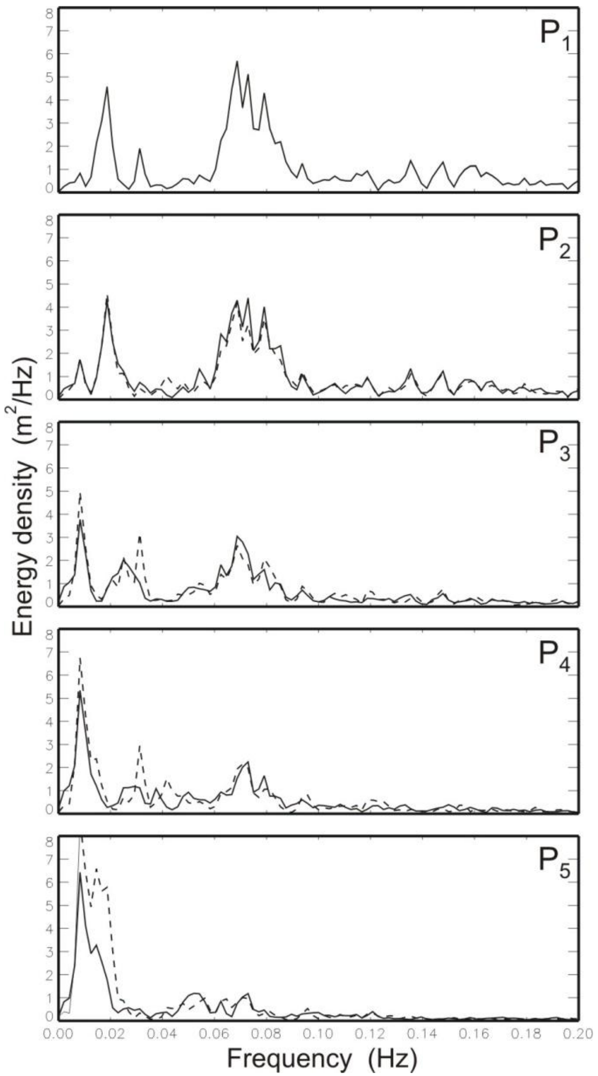


Bottom topography and pressure sensor locations

- Δx=0.4 m, Δt=0.025 s, f_r=0.015



Comparison with field data



Sea-swell frequencies:
$f \in [0.05, 0.2 \text{ Hz}]$

Infragravity frequencies:
$f \in [0.004, 0.05 \text{ Hz}]$

Comparison between observed (solid line) and predicted (dashed line) sea surface elevation density spectra at sensors P1-5; t=th

1D cross-shore mean flow equations

periodic waves

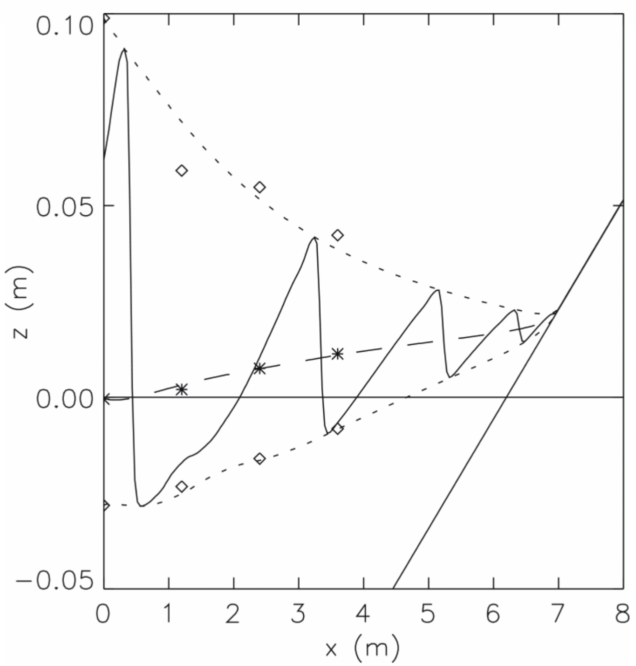
$$\frac{\partial \bar{h}}{\partial t} + \frac{\partial \bar{h}\bar{u}}{\partial x} = -\frac{\partial \overline{\tilde{\zeta}\tilde{u}}}{\partial x}$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + g\frac{\partial \bar{\zeta}}{\partial x} = \mathcal{D} - \frac{\partial \tilde{J}}{\partial x}$$

$$\mathcal{D} = \frac{g}{4c_b T}\frac{(h_2 - h_1)^3}{h_2 h_1}$$

$$\tilde{J} = \frac{1}{2}\overline{\tilde{u}^2}$$

$$\frac{\partial \bar{\zeta}}{\partial x} = \frac{1}{g}\left(\mathcal{D} - \frac{\partial \frac{1}{2}\overline{u^2}}{\partial x}\right)$$

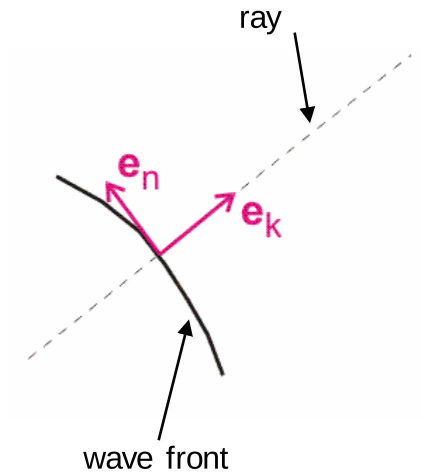


Bonneton (2007)

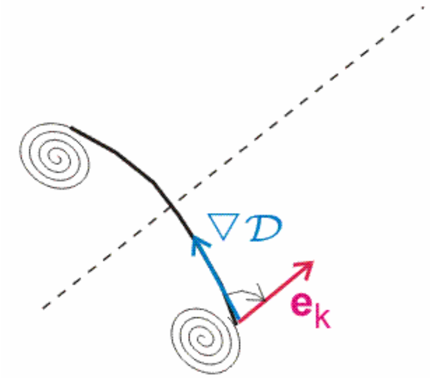
2D mean flow equations

$$\frac{\partial \bar{h}}{\partial t} + \nabla.(\bar{h}\bar{\mathbf{u}}) = -\nabla.\tilde{M}$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}}.\nabla)\bar{\mathbf{u}} + g\nabla\bar{\zeta} = \mathcal{D}\mathbf{e}_k - \nabla\tilde{J} - \overline{\tilde{\omega}(\tilde{\mathbf{u}} \wedge \mathbf{e}_z)}$$



$$\frac{\partial \bar{\omega}}{\partial t} + \nabla.\left(\bar{\omega}\bar{\mathbf{u}} + \overline{\tilde{\omega}\tilde{\mathbf{u}}}\right) = \nabla\mathcal{D} \wedge \mathbf{e}_k$$

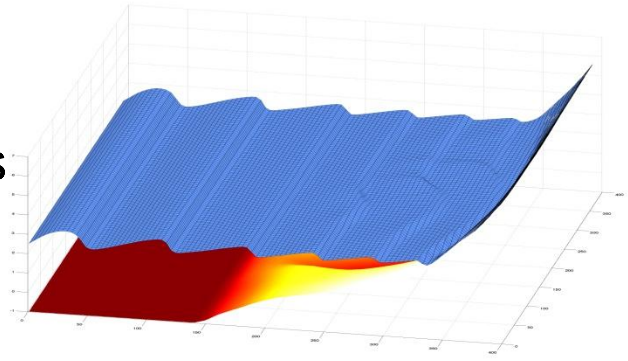


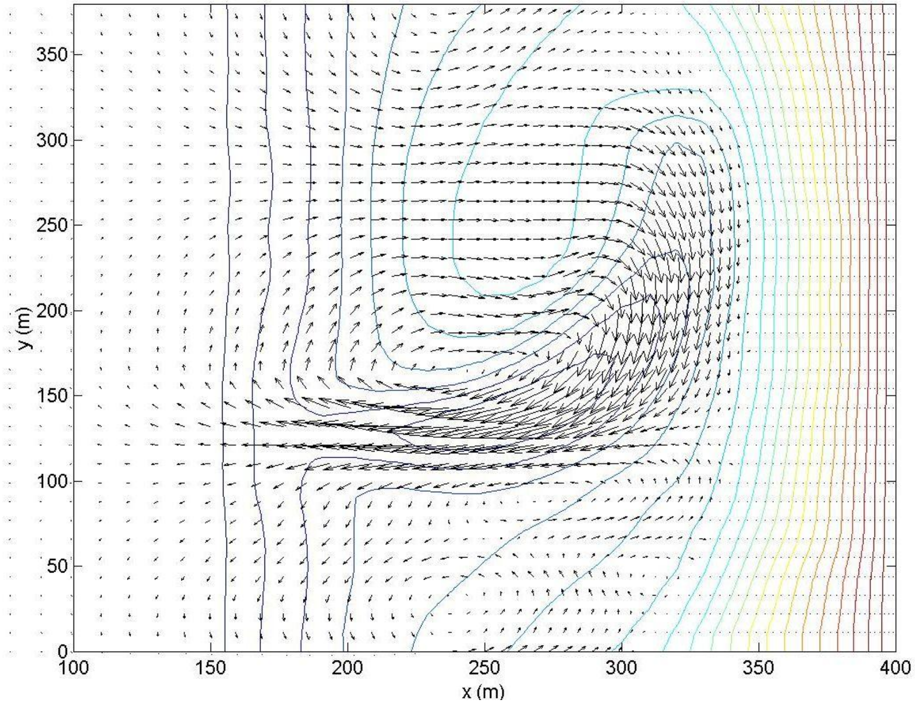
$$\mathcal{D} = \frac{g}{4c_bT}\frac{(h_2 - h_1)^3}{h_2h_1}$$

Wave-induced circulation

High-order well-balanced shock-capturing methods

⇨ SURF_WB model





Marche et Bonneton (2006)

IV – conclusions and perspectives

- longshore non-uniformity of wave breaking (wave energy dissipation) generates vertical vorticity (see also Peregrine (1999) and Bühler (2000))

- wave energy dissipation in the inner surf zone is well reproduced by the shock-wave solutions of the Saint Venant equations

- vorticity equation for wave-induced currents (without ad hoc parametrization):

$$\frac{\partial \bar{\omega}}{\partial t} + \nabla . \left(\bar{\omega} \bar{\mathbf{u}} + \overline{\tilde{\omega} \tilde{\mathbf{u}}} \right) = \nabla \mathcal{D} \wedge \mathbf{e}_k \qquad \mathcal{D} = \frac{g}{4 c_b T} \frac{(h_2 - h_1)^3}{h_2 h_1}$$

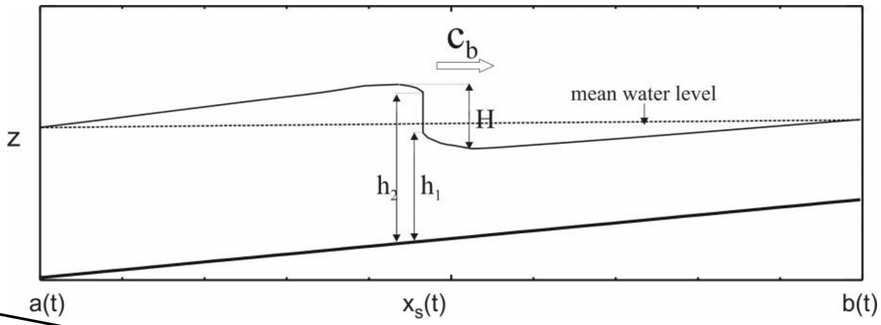
- high-order well-balanced shock capturing models (e.g. SURF_WB) are required to accurately compute wave-induced vorticity

- Green Naghdi extension of shock-capturing Saint Venant approaches

Thank you for your attention

Conditions de saut et dissipation d'énergie

$$E = \int_{a(t)}^{b(t)} \mathcal{E}\, dx \qquad \mathcal{E} = \frac{1}{2}\rho h u^2 + \frac{1}{2}\rho g(h^2 - d^2)$$

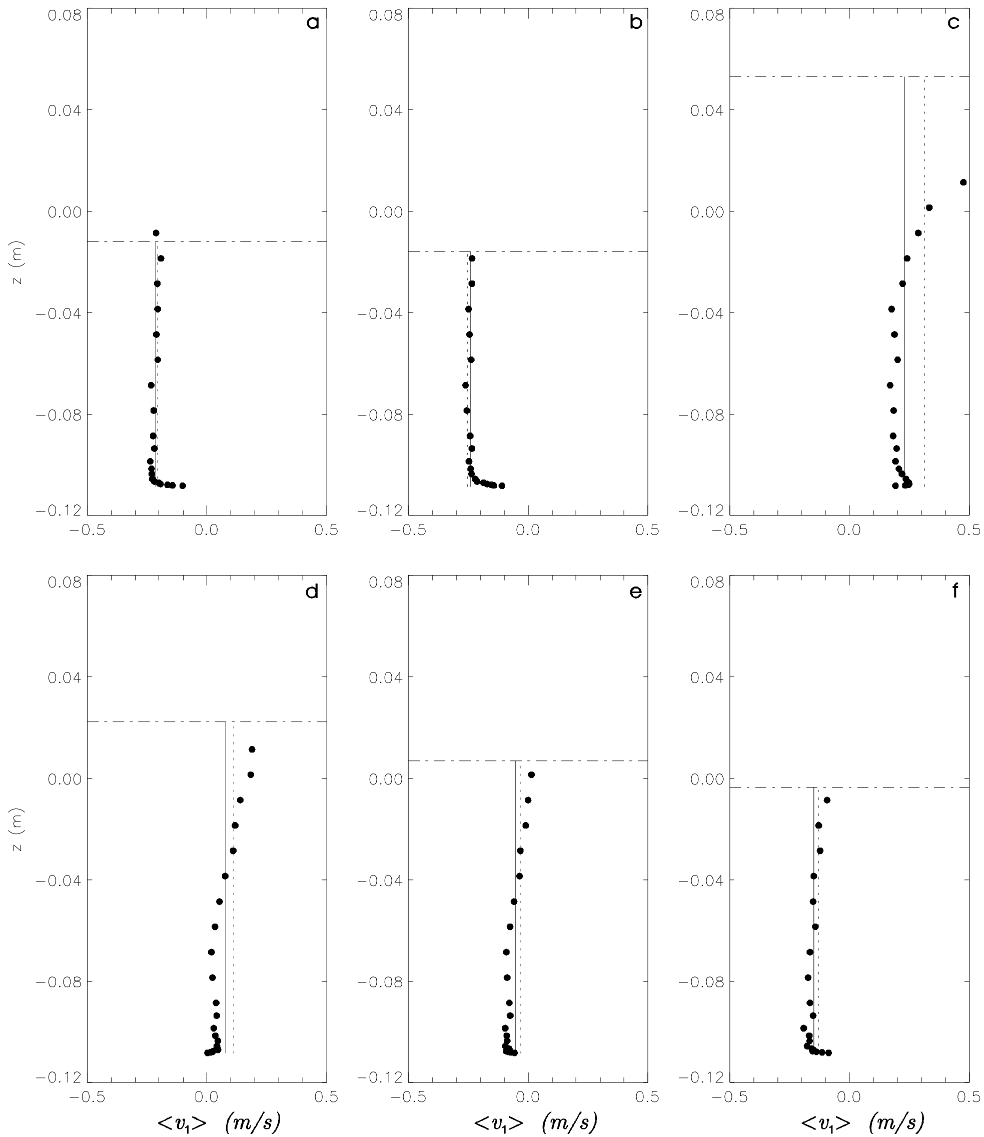


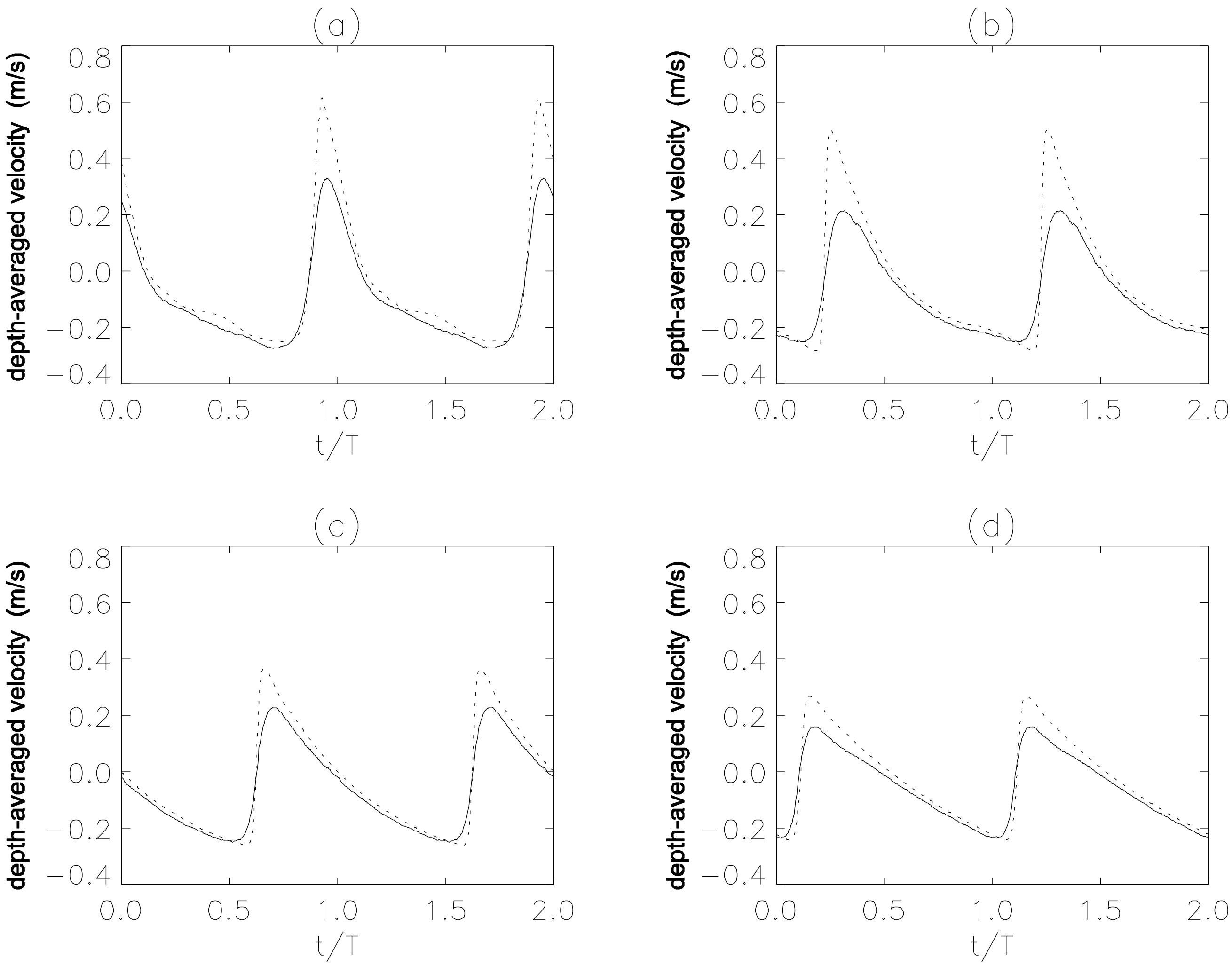
$$D_b = -\frac{dE}{dt} + W$$

$$D_b = -\int_{a(t)}^{x_s(t)} \frac{\partial \mathcal{E}}{\partial t}\, dx - \int_{x_s(t)}^{b(t)} \frac{\partial \mathcal{E}}{\partial t}\, dx - [u\mathcal{E}]_a^b + c_b[\mathcal{E}] - [\frac{1}{2}\rho g h^2 u]_a^b + \int_{a(t)}^{b(t)} \frac{1}{2}\rho f_r |u|^3\, dx$$

RWR ⇨ $D_b = 0 \qquad \frac{\partial \mathcal{E}}{\partial t} + \frac{\partial \mathcal{F}}{\partial x} = -\frac{1}{2}\rho f_r |u|^3 \qquad \mathcal{F} = \rho h u\, (\frac{1}{2}u^2 + g(h-d))$

shock ⇨ $D_b = -[\mathcal{F}] + c_b[\mathcal{E}] = \frac{g}{4}(h_2 - h_1)^3 \Big(\frac{g(h_2+h_1)}{2h_1h_2}\Big)^{\frac{1}{2}}$ (Stoker (1957))


a
b
c
d
e
f
z (m)
z (m)
0.08
0.04
0.00
−0.04
−0.08
−0.12
−0.5
0.0
0.5
<v1> (m/s)
<v1> (m/s)
<v1> (m/s)



(a)
(b)
(c)
(d)
depth-averaged velocity (m/s)
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0.0
0.5
1.0
1.5
2.0
t/T