UNIVERSITÉ DE BORDEAUX

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A simple and accurate nonlinear method for recovering the surface wave elevation from pressure measurements

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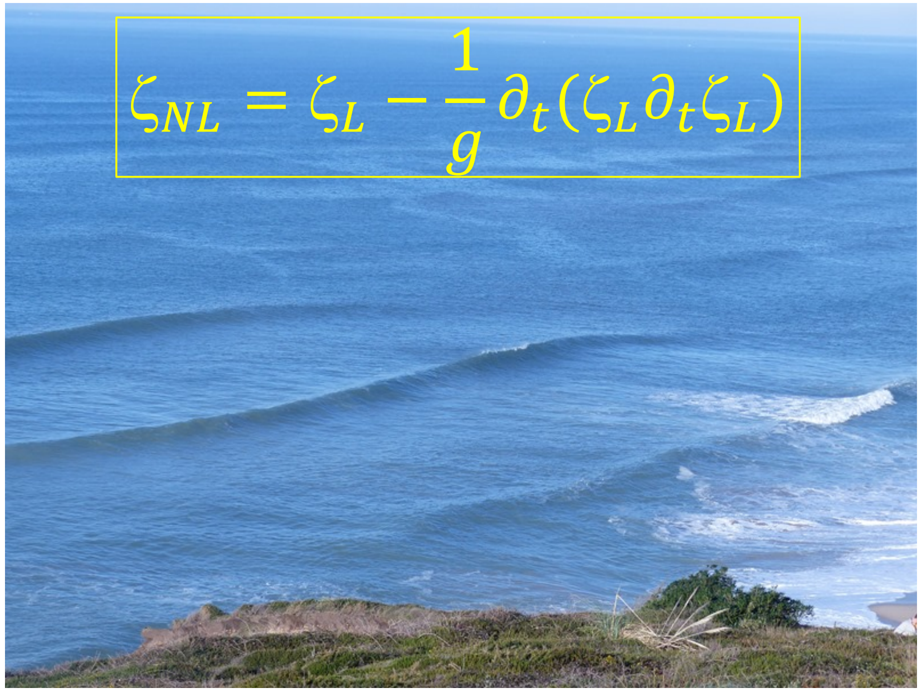
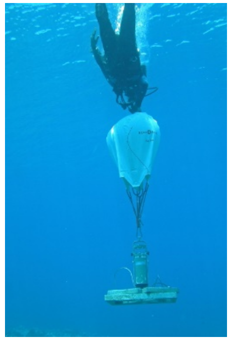
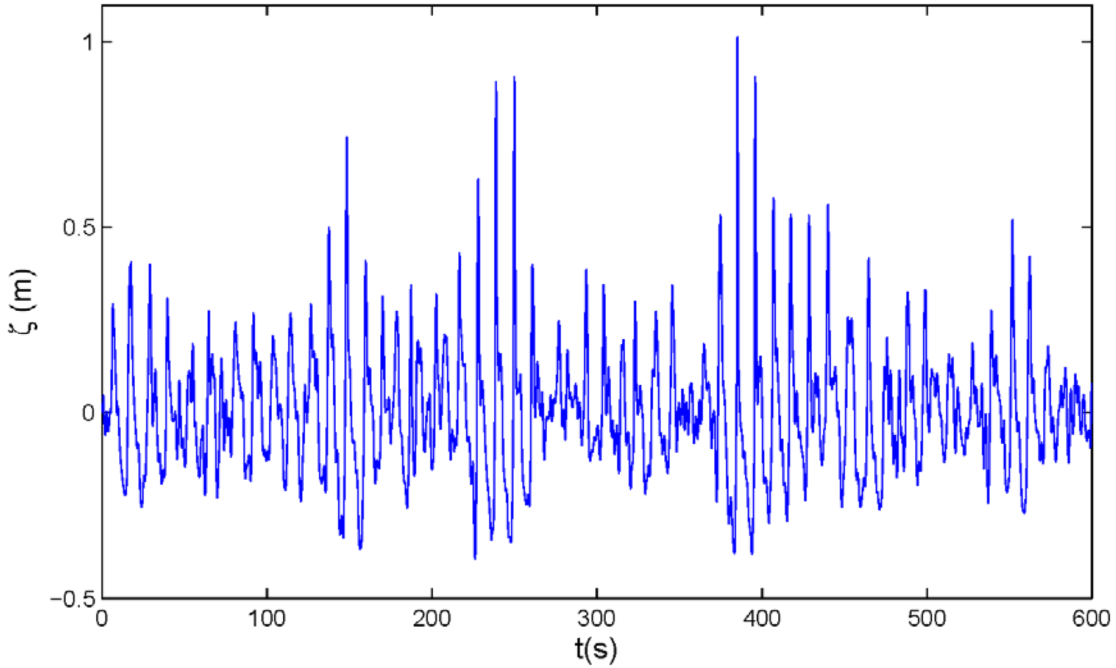




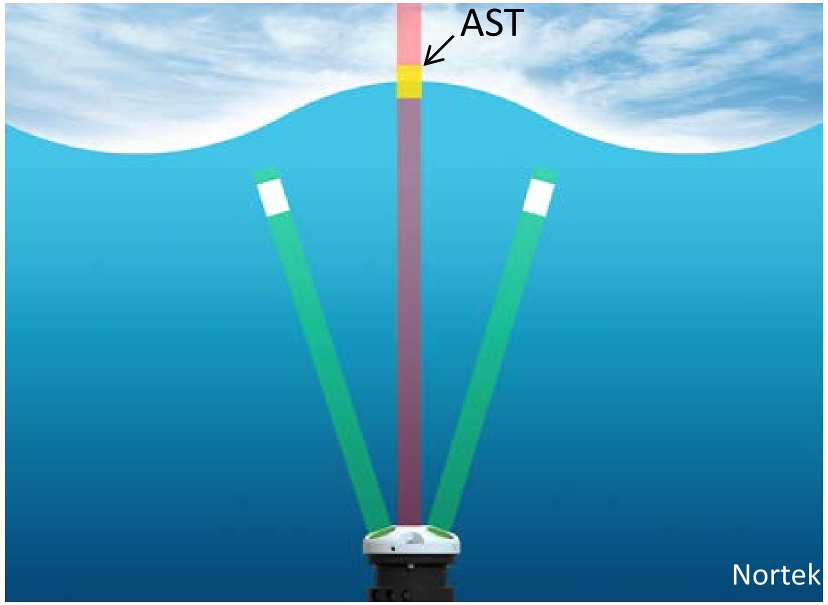
Accurate measurements of surface wave elevation ζ are crucial for many coastal applications:



- ❑ **wave overtopping and submersion**
- ❑ **navigation and platform safety**
- ❑ **wave-induced sediment transport**

Direct measurement of the surface elevation → **highly accurate measurement**

Acoustic surface tracking



Lidar scanning

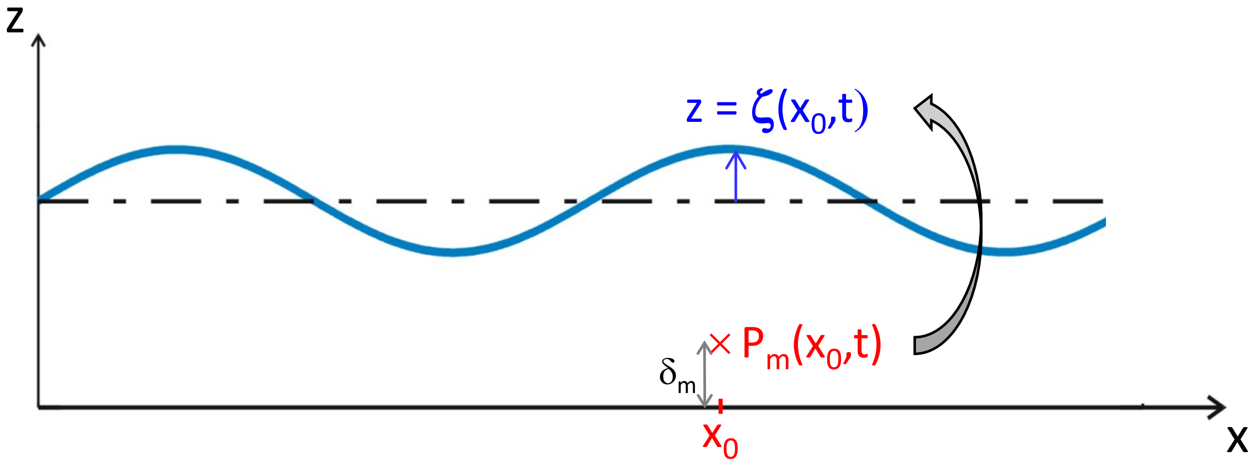
Martins et al 2018

- ✓ expensive
- ✓ difficult to deploy and fragile
- ✓ AST sensitive to air bubbles and turbidity
- ✓ lidar requires the presence of nearshore structures

Pressure sensors are still a very useful tool for measuring waves

Pressure sensor

Ocean Sensor Systems



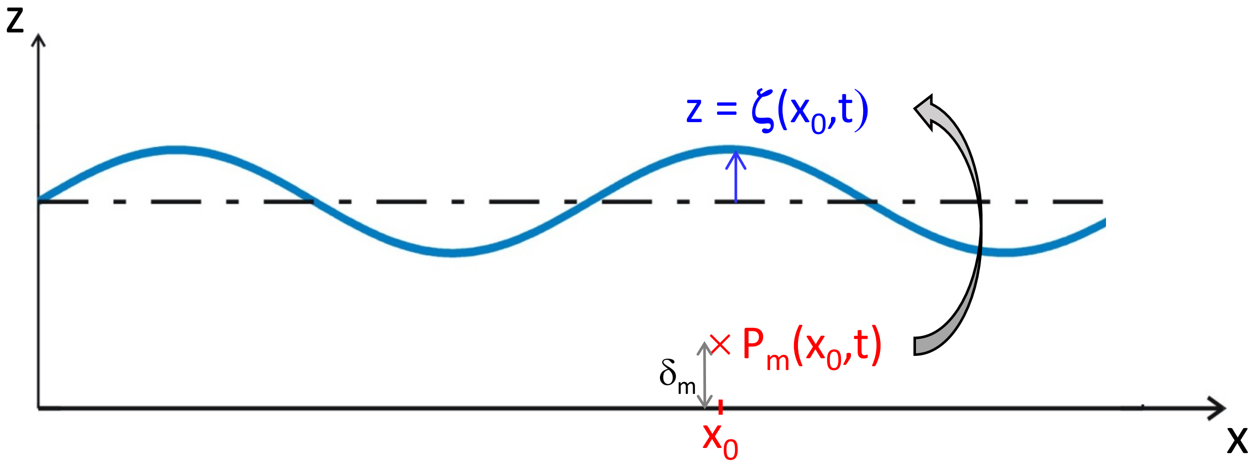
- ✓ cheap
- ✓ easy to deploy
- ✓ robust (storms, bottom trawling, …)
- ✓ not sensitive to air bubbles and turbidity

not a direct measurement of ζ → <u>methods for recovering ζ</u>

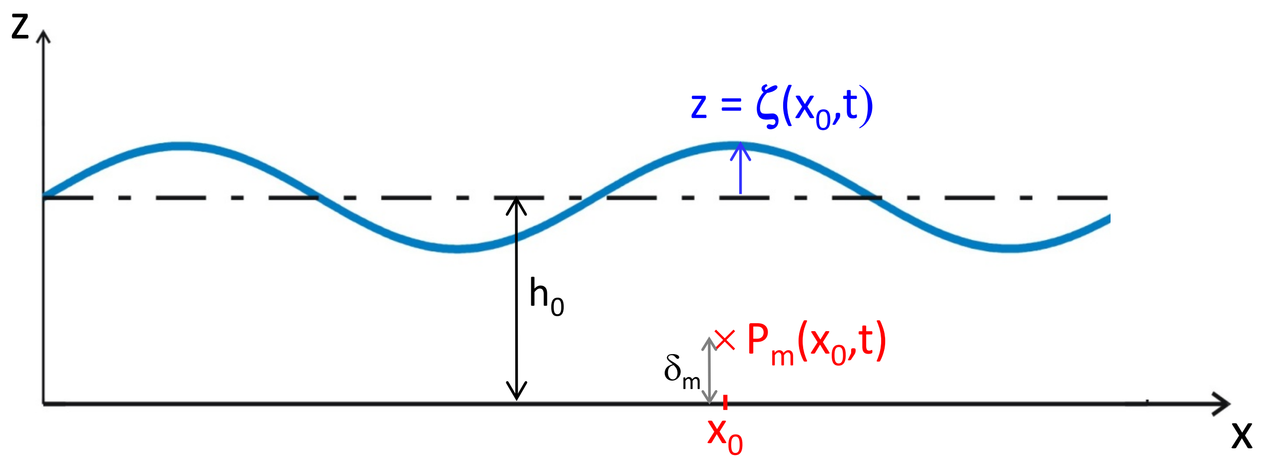
Commonly used reconstruction methods and their limitations

Pressure sensor

Ocean Sensor Systems



long waves: tsunamis, tides, … → **hydrostatic reconstruction**



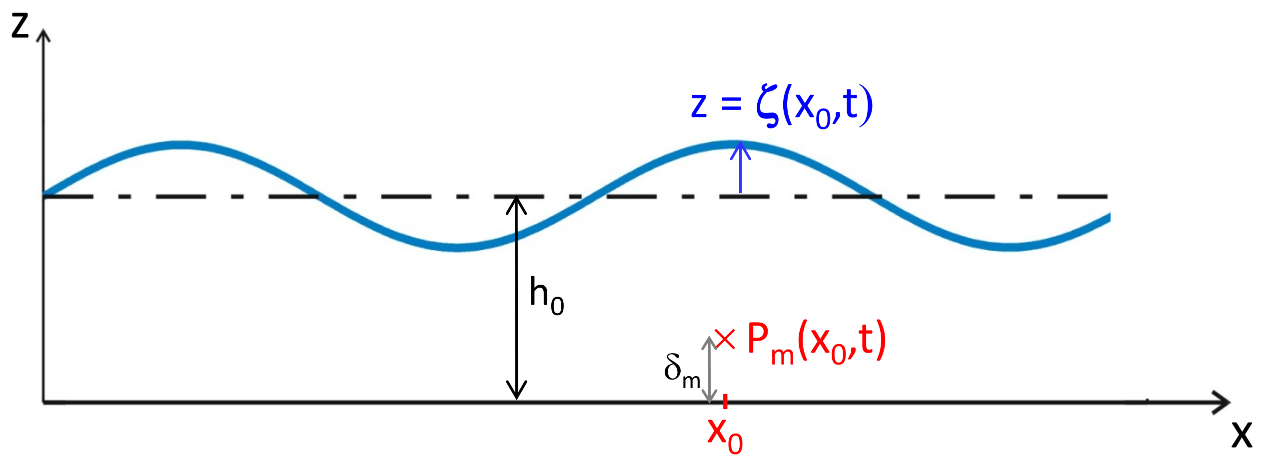
$$\frac{\partial P}{\partial z} = -\rho_0 g \quad \Rightarrow \quad h_H(x_0, t) = \frac{P_m - P_a}{\rho_0 g} + \delta_m$$

$$\zeta_H = \frac{P_m - P_a}{\rho_0 g} + \delta_m - h_0$$

swell, wind-generated waves $\rightarrow$ non-hydrostatic reconstruction

swell, wind-generated waves $\rightarrow$ non-hydrostatic reconstruction

recover the wave field by means of a transfer function based on linear theory   ***" transfer function method "***



pressure response factor

$$\tilde{\zeta}_L(\omega) = \frac{\cosh(h_0|k|)}{\cosh(\delta_m|k|)}\tilde{\zeta}_H(\omega)$$

$$\zeta_H = \frac{P_m - P_a}{\rho_0 g} + \delta_m - h_0$$

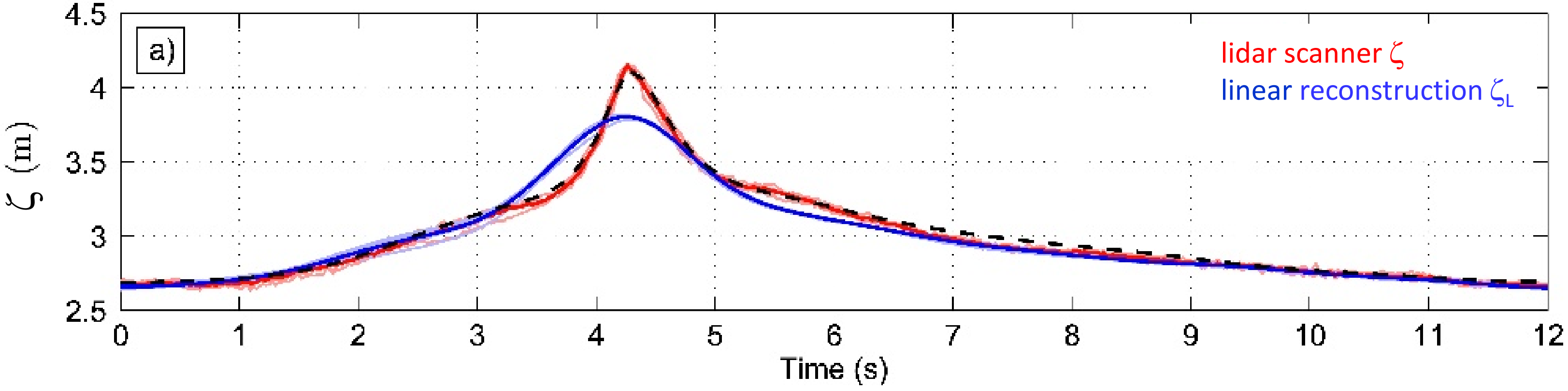
$$\omega^2 = g|k|\tanh(h_0|k(\omega)|)$$

swell, wind-generated waves → non-hydrostatic reconstruction ***" transfer function method "***

- gives reasonable estimates for bulk wave parameters such as H_s

Guza and Thornton 1980, Bishop and Donelan 1987, Tsai et al. 2005, …

- fails to describe the shape of nonlinear nearshore waves



Martins et al. 2017, BARDEX II, h_0=1.17 m, T_p=12.1 s

→ provides a poor description of the peaky and skewed shape of nonlinear waves

→ underestimates the individual wave height by up to 30 %

There is a critical need for nonlinear reconstruction methods

- *Constantin 2012, Deconinck et al. 2012, Olivears et al. 2012, Clamond, 2013*

  → periodic waves of permanent form

- *Oliveras et al. 2012*

  → heuristic approximation for irregular waves

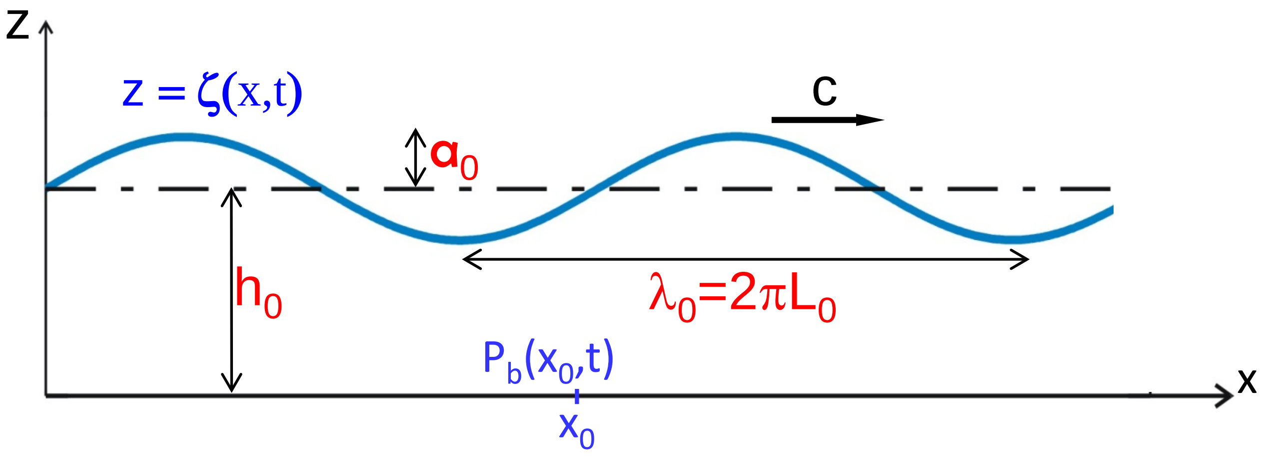
Two nonlinear reconstruction methods for irregular waves in the field

- *Bonneton, Lannes, Martins, Michallet, Coastal Eng. 2018*

  $\rightarrow$ weakly dispersive

- *Bonneton and Lannes, JFM 2017*

  $\rightarrow$ fully dispersive



$$\varepsilon = \frac{a_0}{h_0} = O(1) \qquad \mu = \left(\frac{h_0}{L_0}\right)^2 \lesssim 1$$

$$\sigma = \frac{a_0}{L_0} = \varepsilon\sqrt{\mu} \ll 1$$

Asymptotic expansion of the Euler equations in terms of $\sigma \rightarrow \quad \phi = \phi_0 + \sigma\phi_1 + O(\sigma^2)$

$$\zeta_{NL} = \zeta_L - \sqrt{\mu}\sigma\, \partial_t(\zeta_L \partial_t \zeta_L)$$

Bonneton and Lannes (JFM 2017)

$$\hat{\zeta}_L(k) = \cosh\left(\sqrt{\mu}|k|\right)\hat{\zeta}_H(k)$$

In variables with dimension

$$\zeta_{NL} = \zeta_L - \frac{1}{g}\partial_t(\zeta_L \partial_t \zeta_L)$$

$$\hat{\zeta}_L(k) = \cosh(h_0|k|)\,\hat{\zeta}_H(k)$$

Waves in shallow water $(\mu \ll 1)$

Asymptotic expansion of the Euler equations in terms of μ $\rightarrow$ $\phi = \phi_0 + \mu\phi_1 + O(\mu^2)$

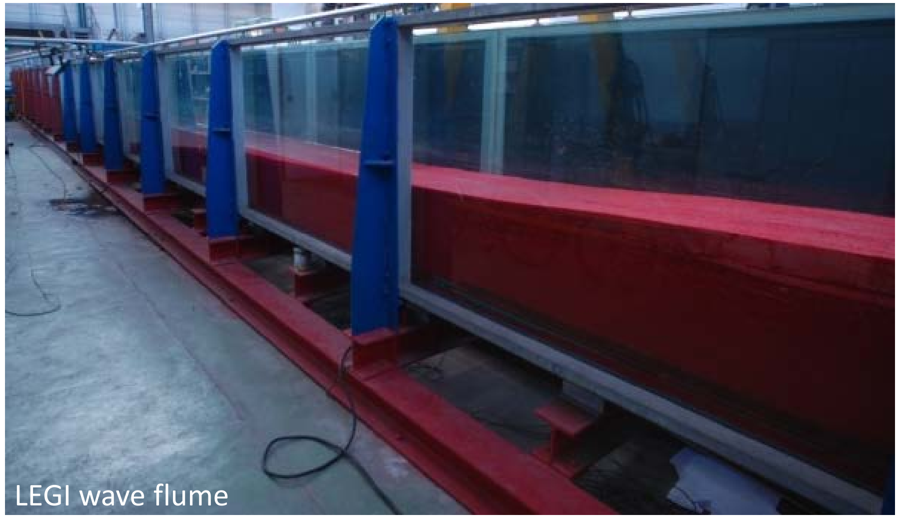
$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g}\partial_t(\zeta_{SL}\partial_t\zeta_{SL})$$

Bonneton et al., Coastal Eng. 2018

$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g}\partial_t^2\zeta_H$$

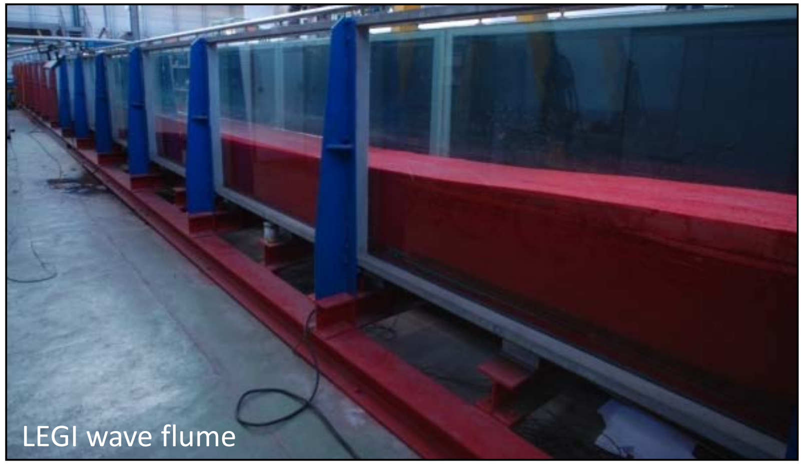
Applications to laboratory and field data

Laboratory experiments



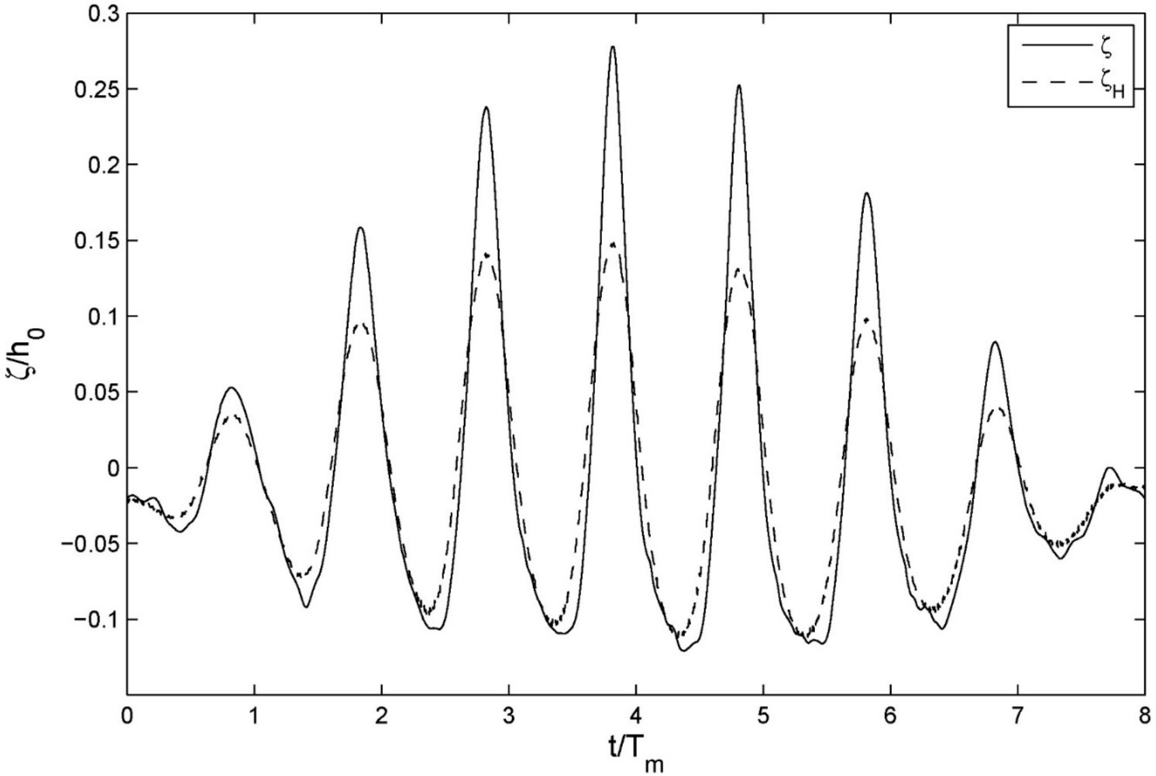
Field measurements

LEGI wave flume experiments, *Michallet et al. 2017*



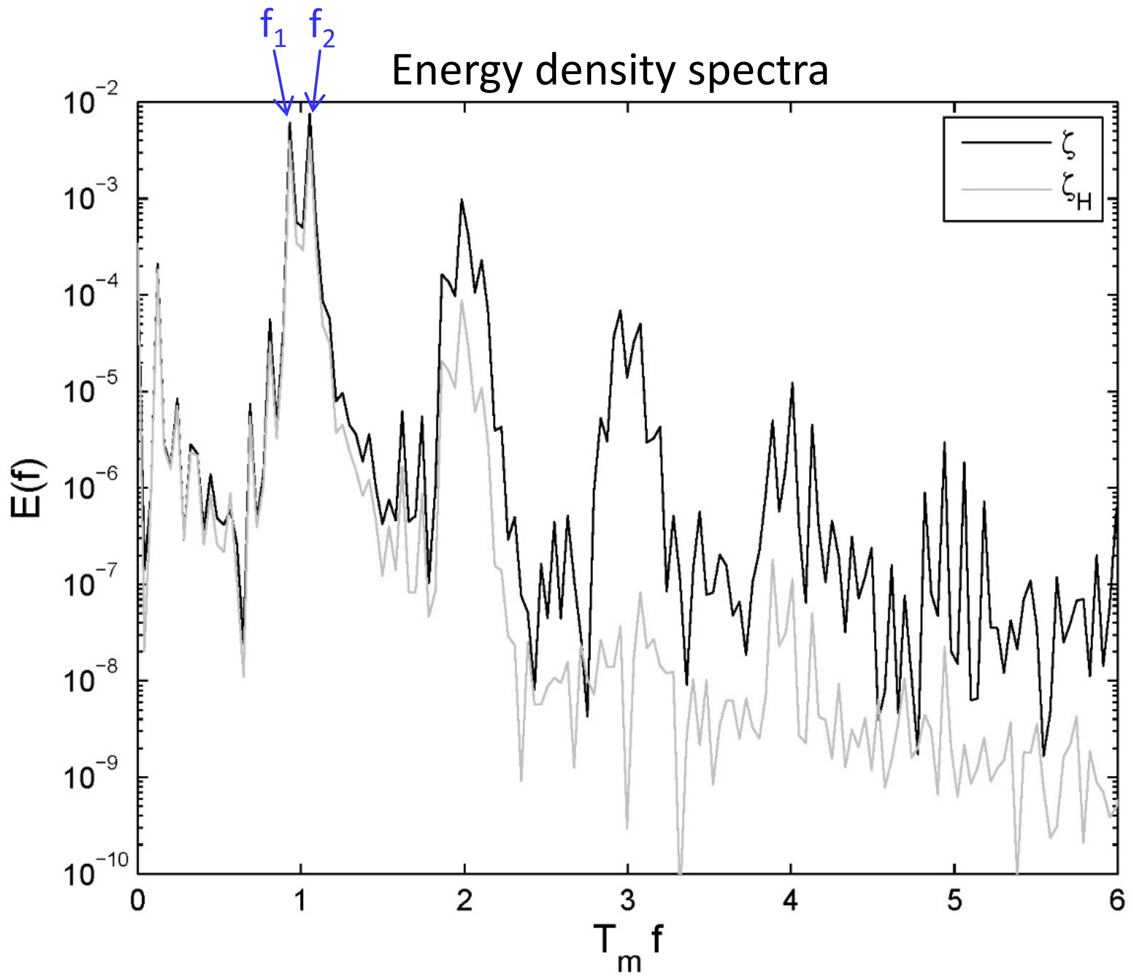
36 m long, 0.55 m wide

bichromatic waves propagating over a gently sloping (1/20) movable bed



ζ and P_m were synchronously measured in the shoaling zone, at 18.5m from the wave maker; h_0=0.326 m, T_m=1.7 s, μ=0.53

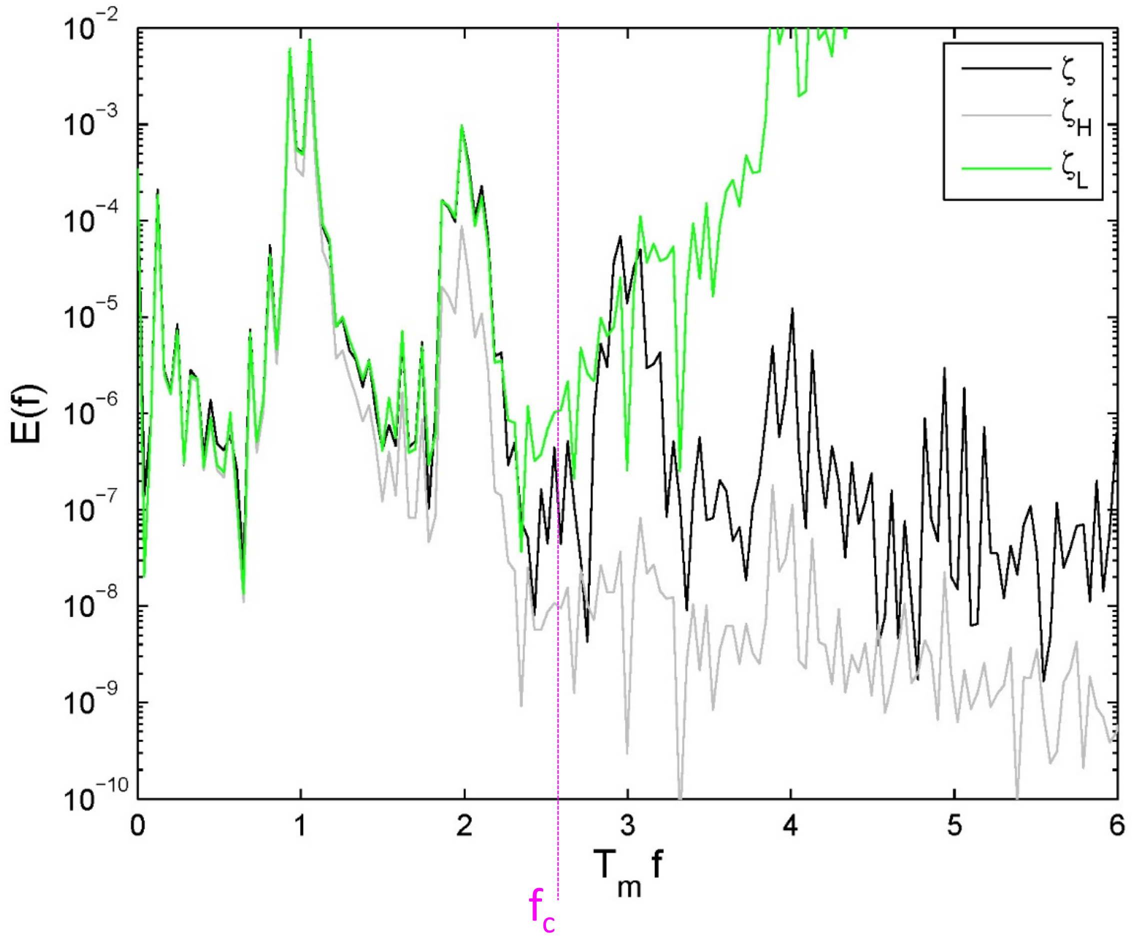
bichromatic waves propagating over a gently sloping movable bed



$$\tilde{\zeta}_L(\omega) = \cosh(h_0|k(\omega)|)\,\tilde{\zeta}_H(\omega)$$

linear reconstruction

$$\omega^2 = g|k|\tanh(h_0|k(\omega)|)$$



$$\tilde{\zeta}_L(\omega) = \cosh(h_0|k(\omega)|)\,\tilde{\zeta}_H(\omega)$$

linear reconstruction

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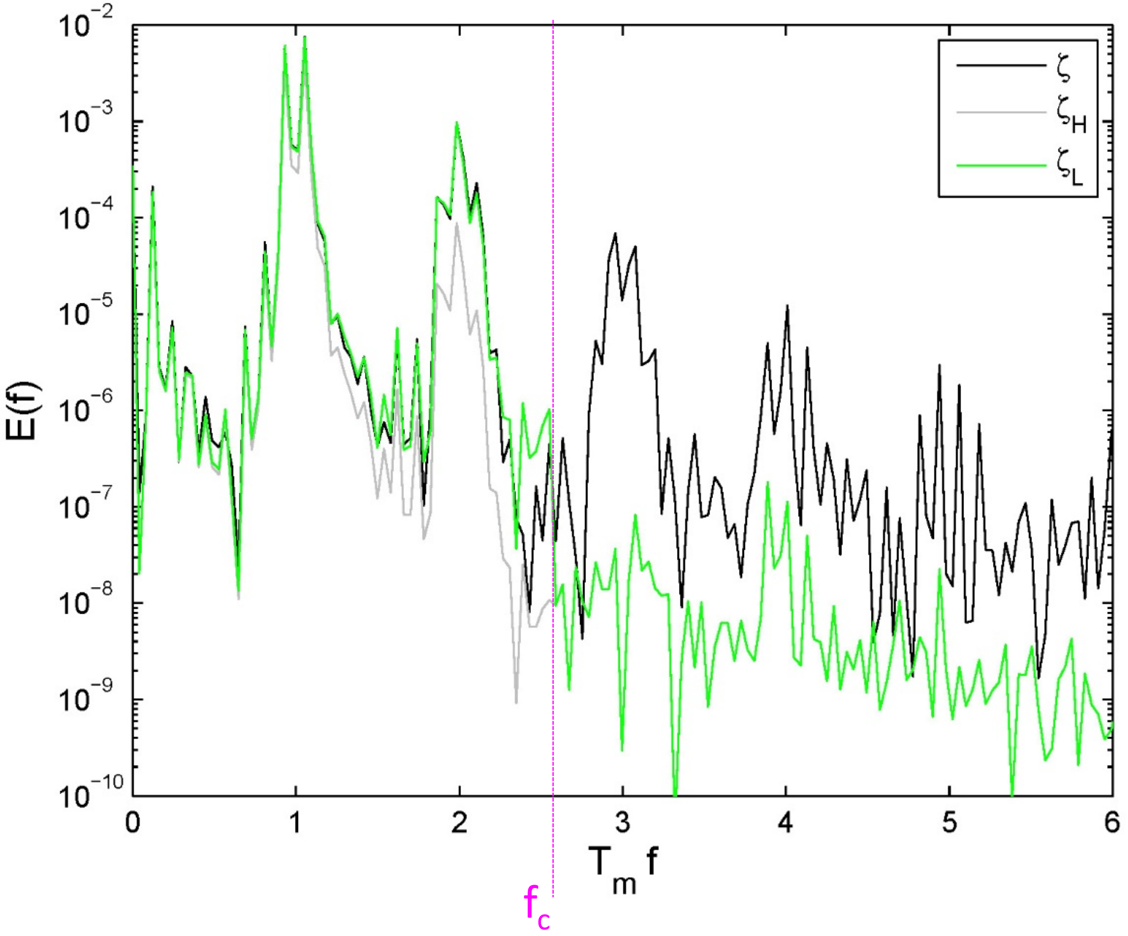


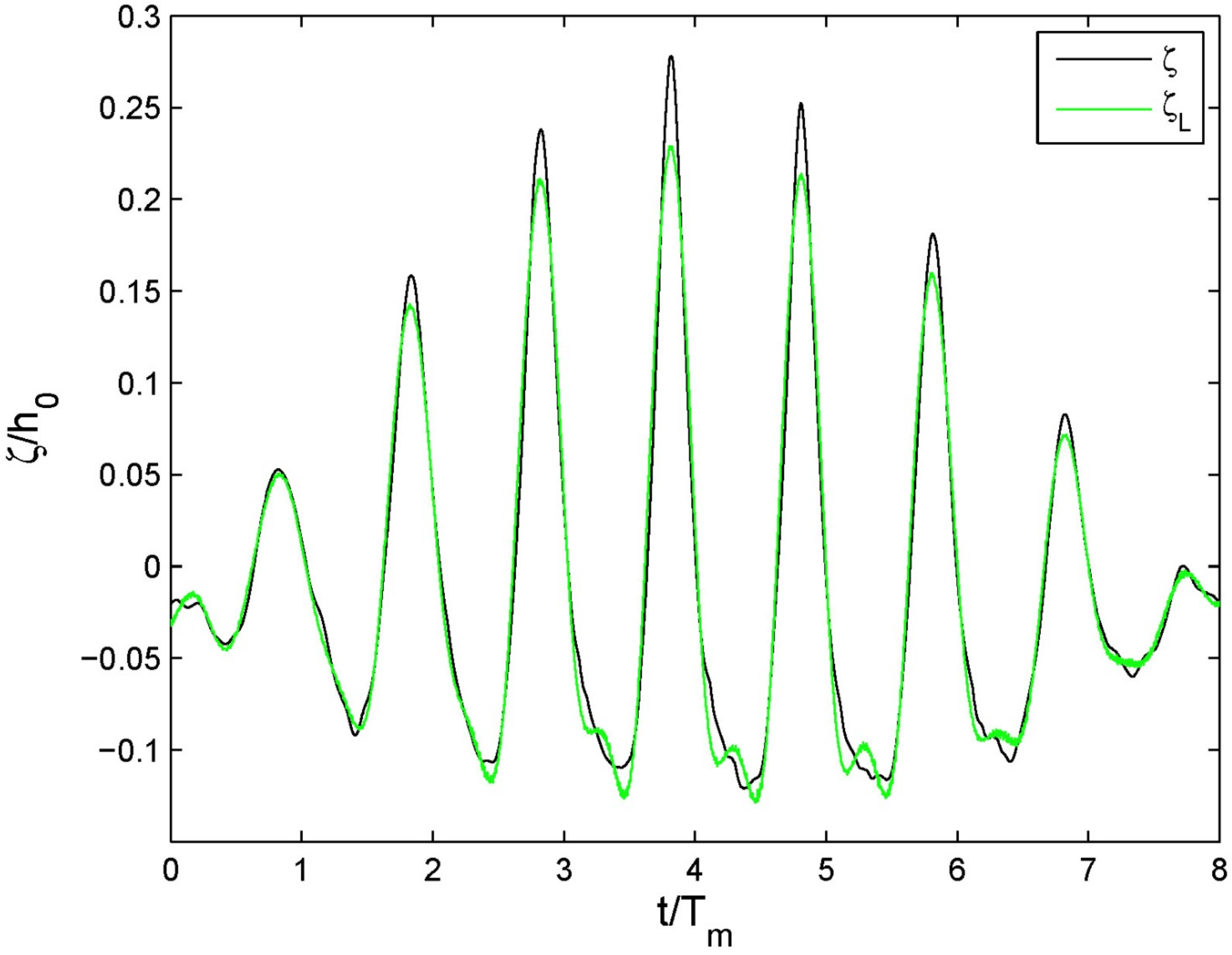
$f > f_c$

$$\tilde{\zeta}_L(\omega) = \tilde{\zeta}_H(\omega)$$

$$\tilde{\zeta}_L(\omega) = \cosh(h_0|k(\omega)|)\,\tilde{\zeta}_H(\omega)$$

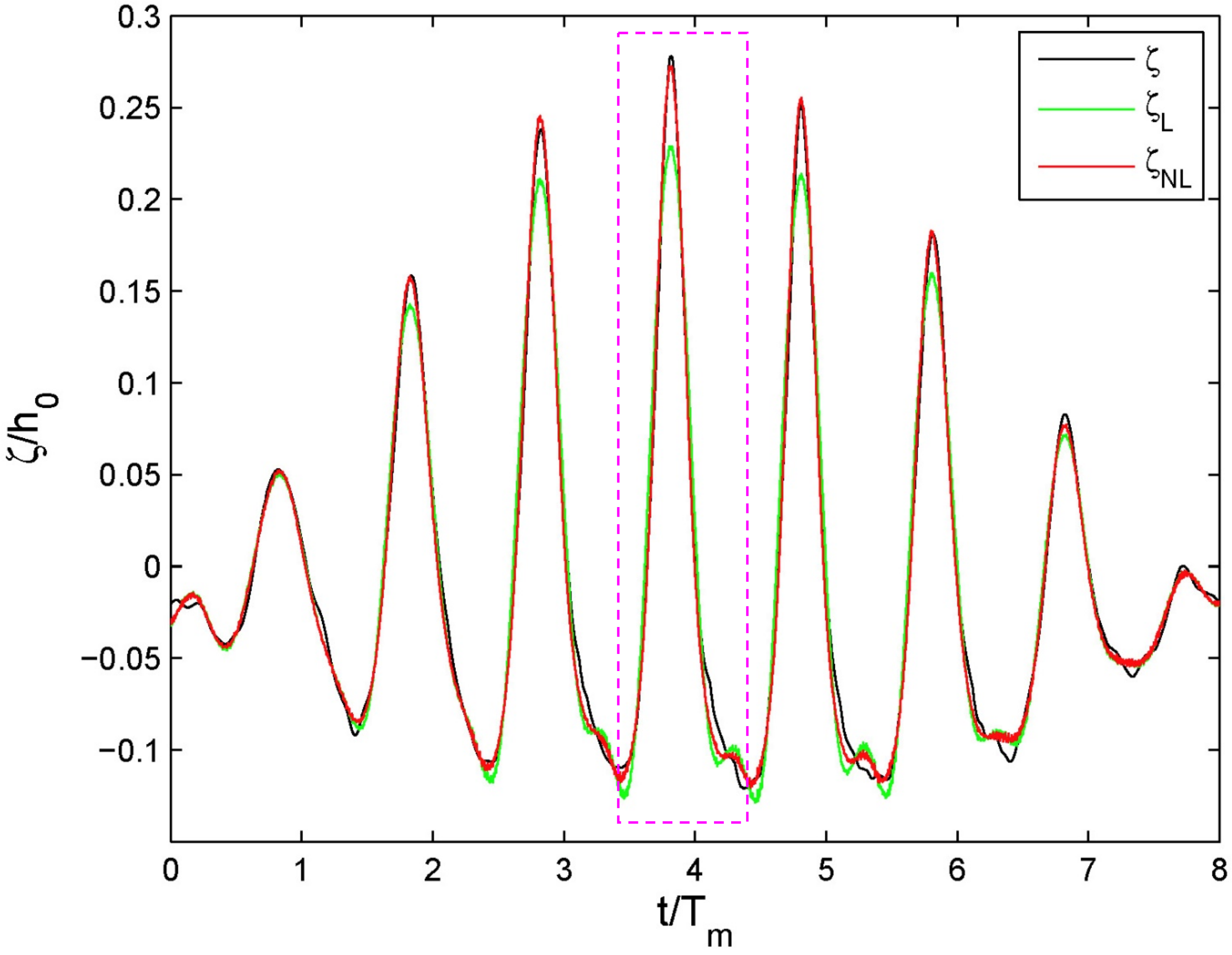
linear reconstruction

$$\omega^2 = g|k|\tanh(h_0|k(\omega)|)$$



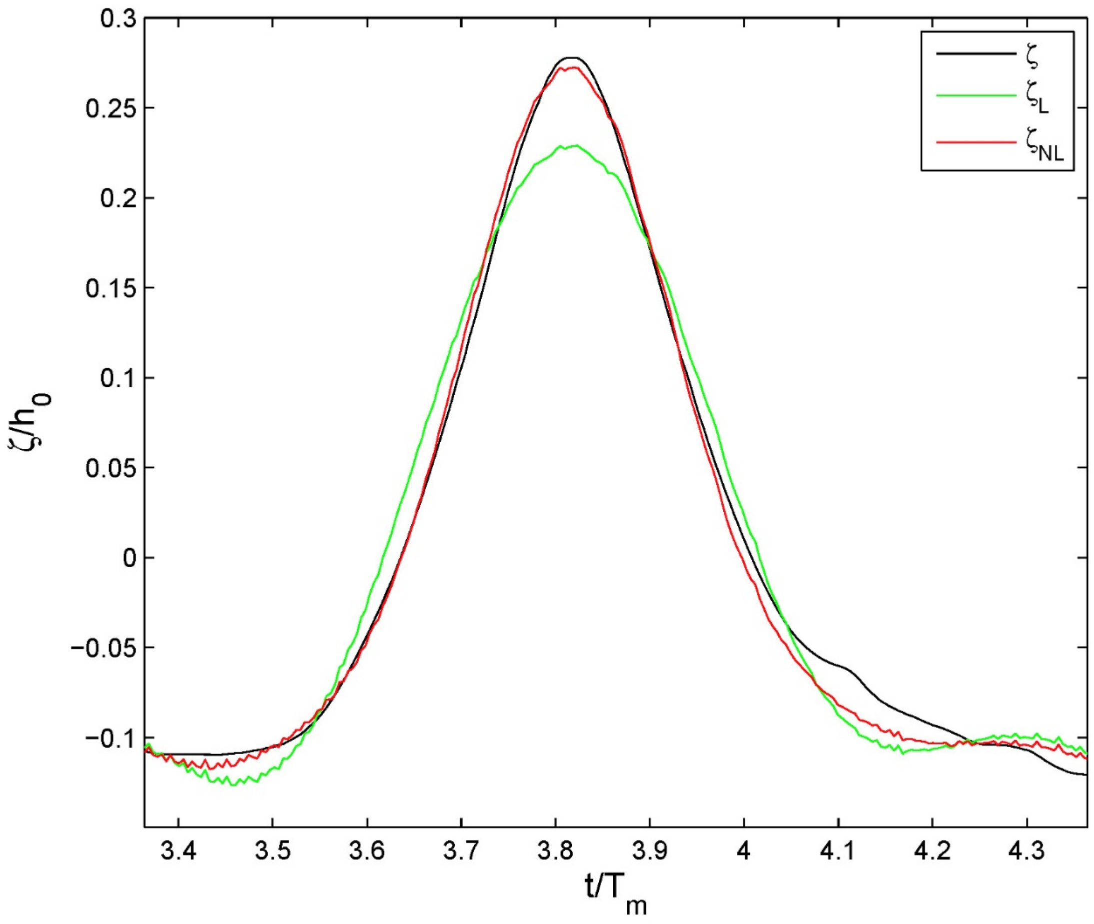
$$\zeta_{NL} = \zeta_L - \frac{1}{g}\partial_t(\zeta_L \partial_t \zeta_L)$$

non-Linear reconstruction



$$\zeta_{NL} = \zeta_L - \frac{1}{g}\partial_t(\zeta_L \partial_t \zeta_L)$$

non-Linear reconstruction

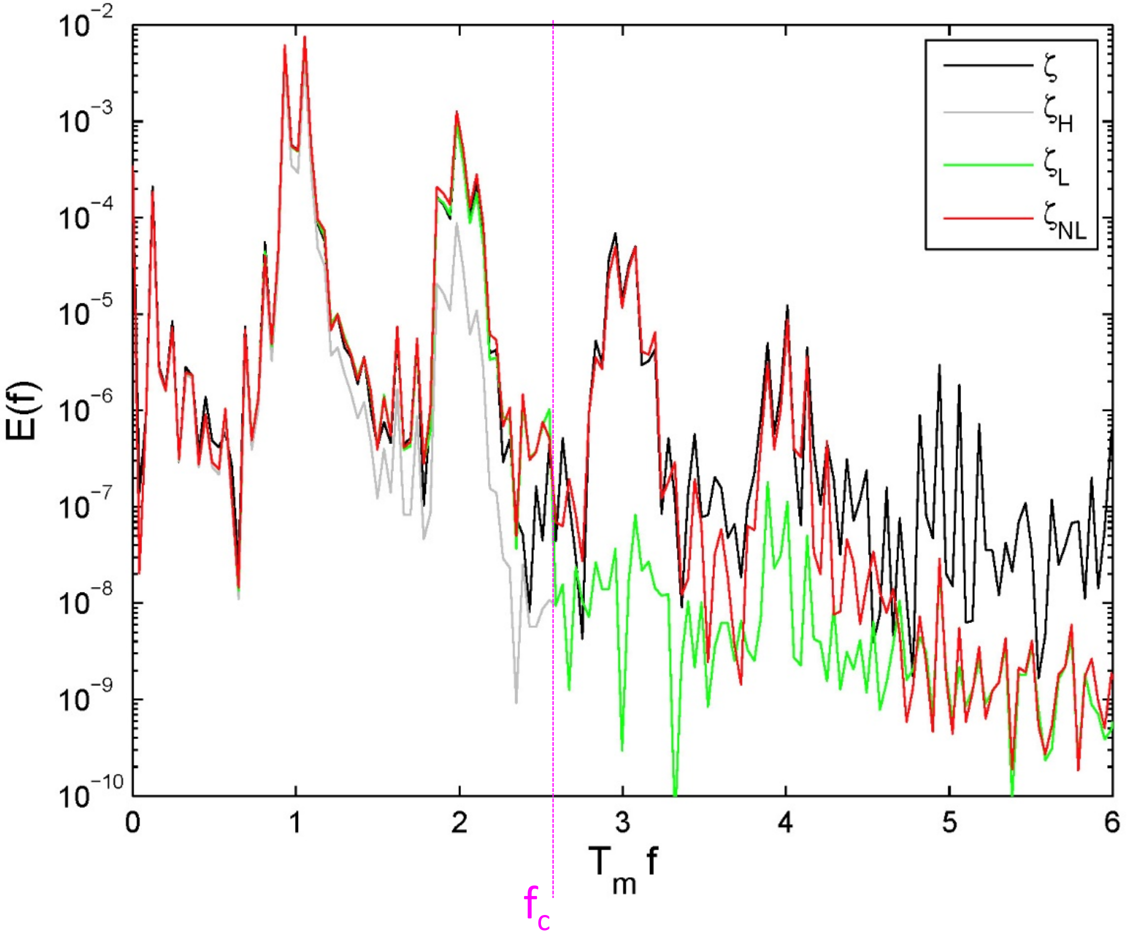


$$S_k = \frac{\langle(\zeta - \langle\zeta\rangle)^3\rangle}{\langle(\zeta - \langle\zeta\rangle)^2\rangle^{3/2}}$$

	ζ direct measurement	ζ_L	ζ_{NL}
S_K	0.93	0.70	0.96
S_K error	—	25%	3%

$$\zeta_{NL} = \zeta_L - \frac{1}{g}\partial_t(\zeta_L \partial_t \zeta_L)$$

non-Linear reconstruction



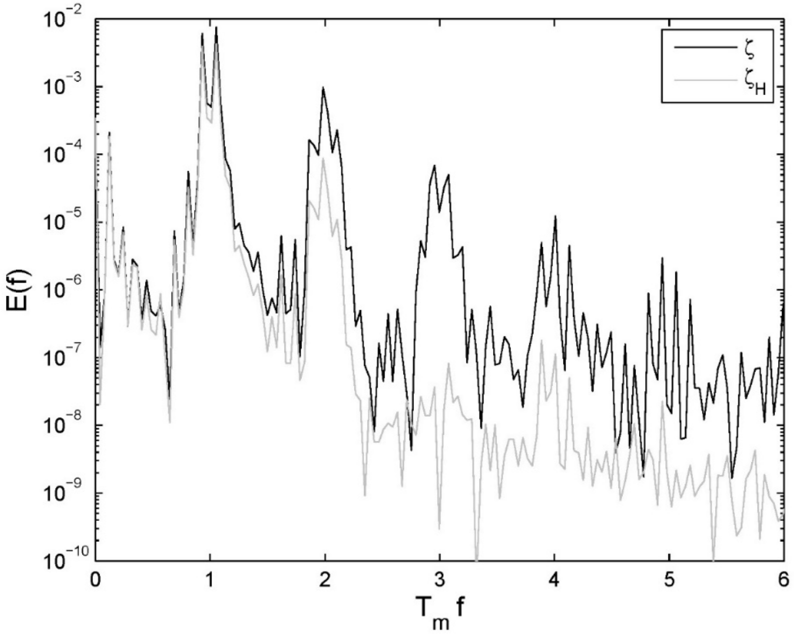
Highest wave nonlinearities generally occur in shallow water close to the onset of breaking

nonlinear weakly dispersive reconstruction

$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g}\,\partial_t\left(\zeta_{SL}\partial_t\zeta_{SL}\right)$$

$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g}\,\partial_t^2\zeta_H$$

- local in time
- no frequency cut-off f_c
- reconstruction of the whole elevation density spectrum
- $\mu < 0.3$

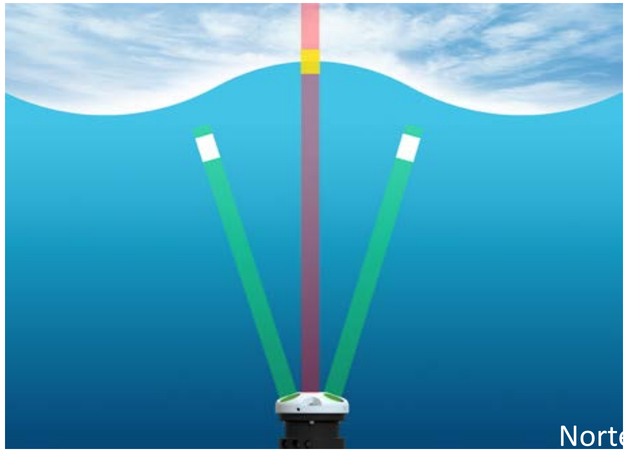


Field campaign (April 13-14, 2017)

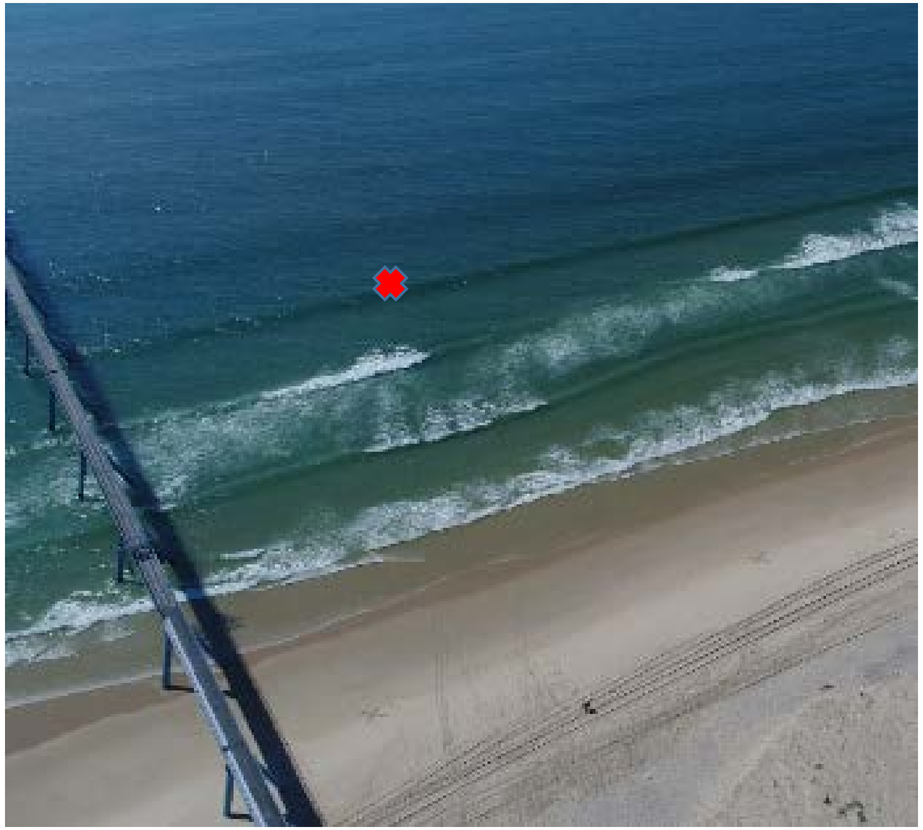
La Salie beach, southern part of the French Atlantic coast

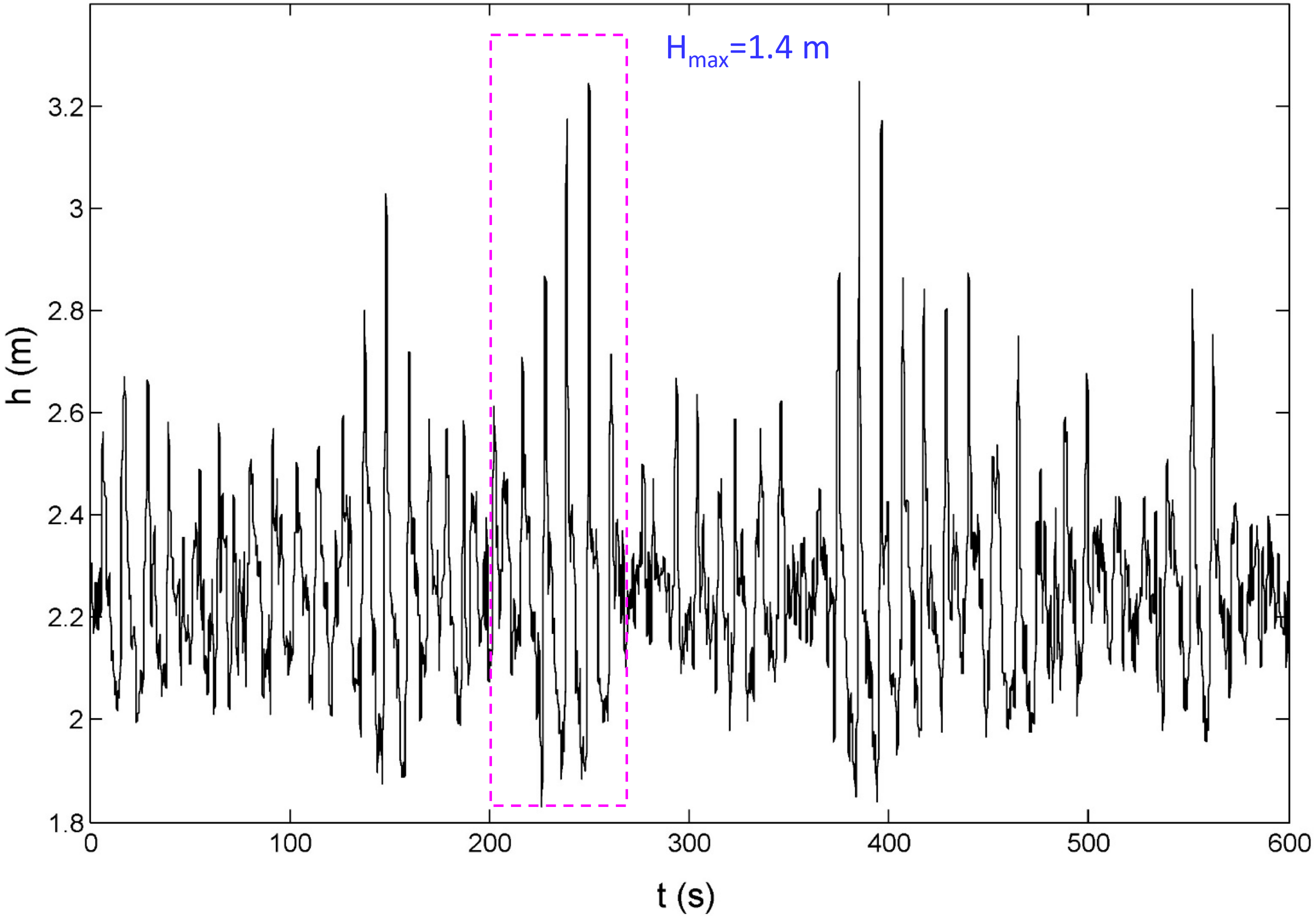
Instruments were deployed at low tide:

- pressure transducers (Ocean Sensor Systems)
- Nortek ADCP, Signature 1000 $\rightarrow$ direct measurement of ζ from the vertical beam of the ADCP



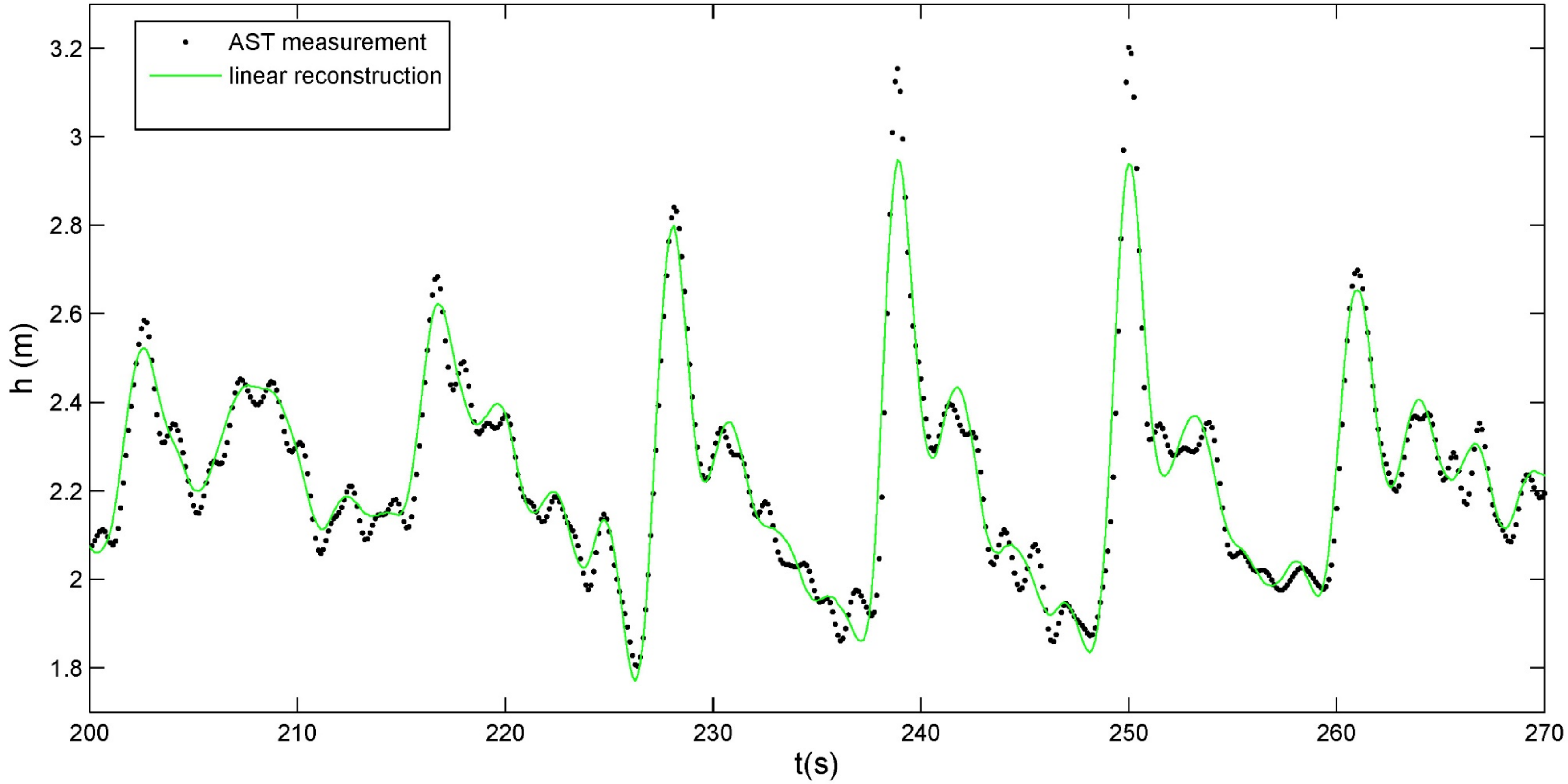
Field campaign → nonlinear waves in the shoaling zone just prior to breaking





h_0=2.25 m, H_s=0.70 m, T_p=11.1 s μ=0.075 → weakly dispersive waves

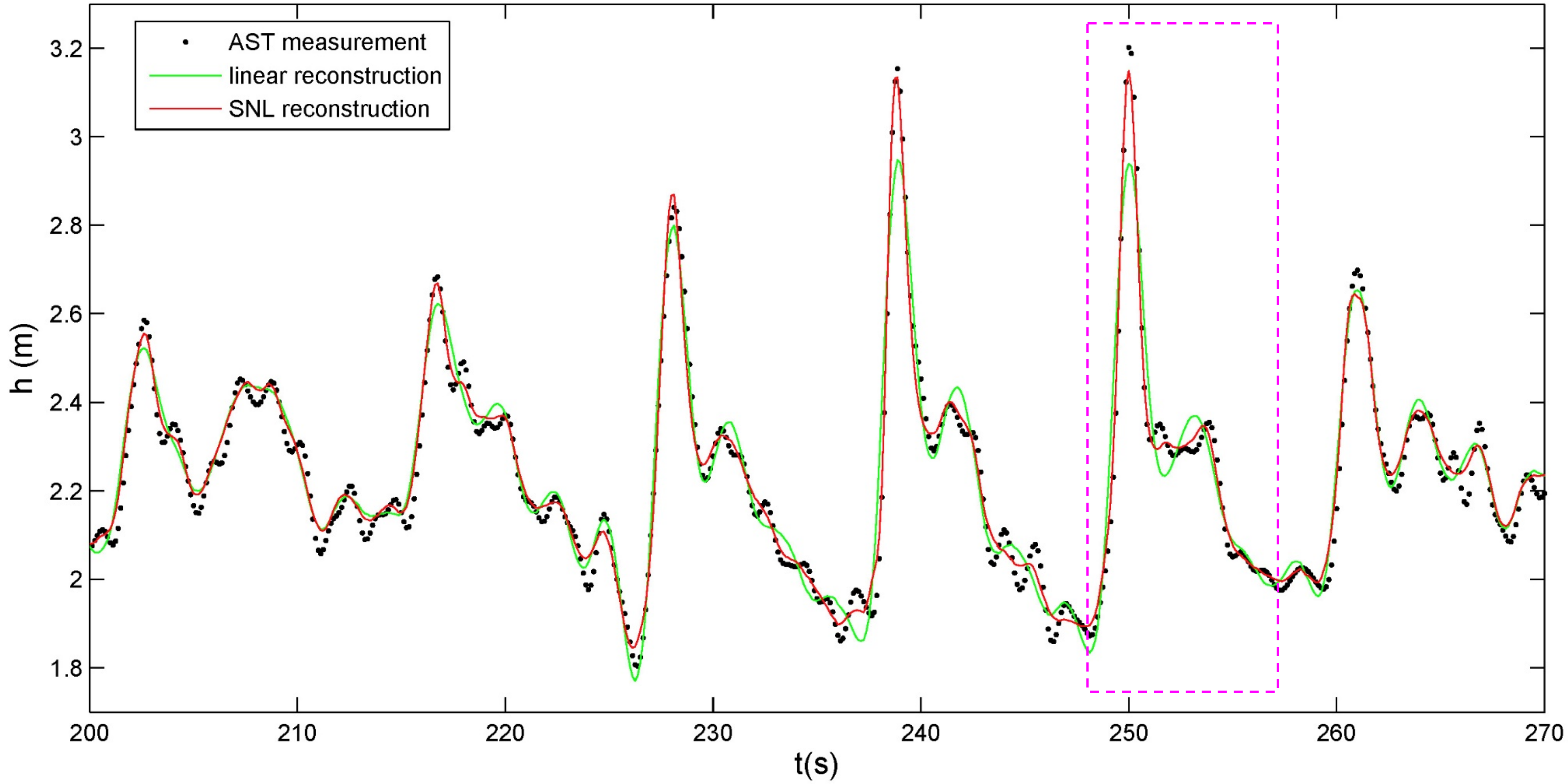
Field campaign



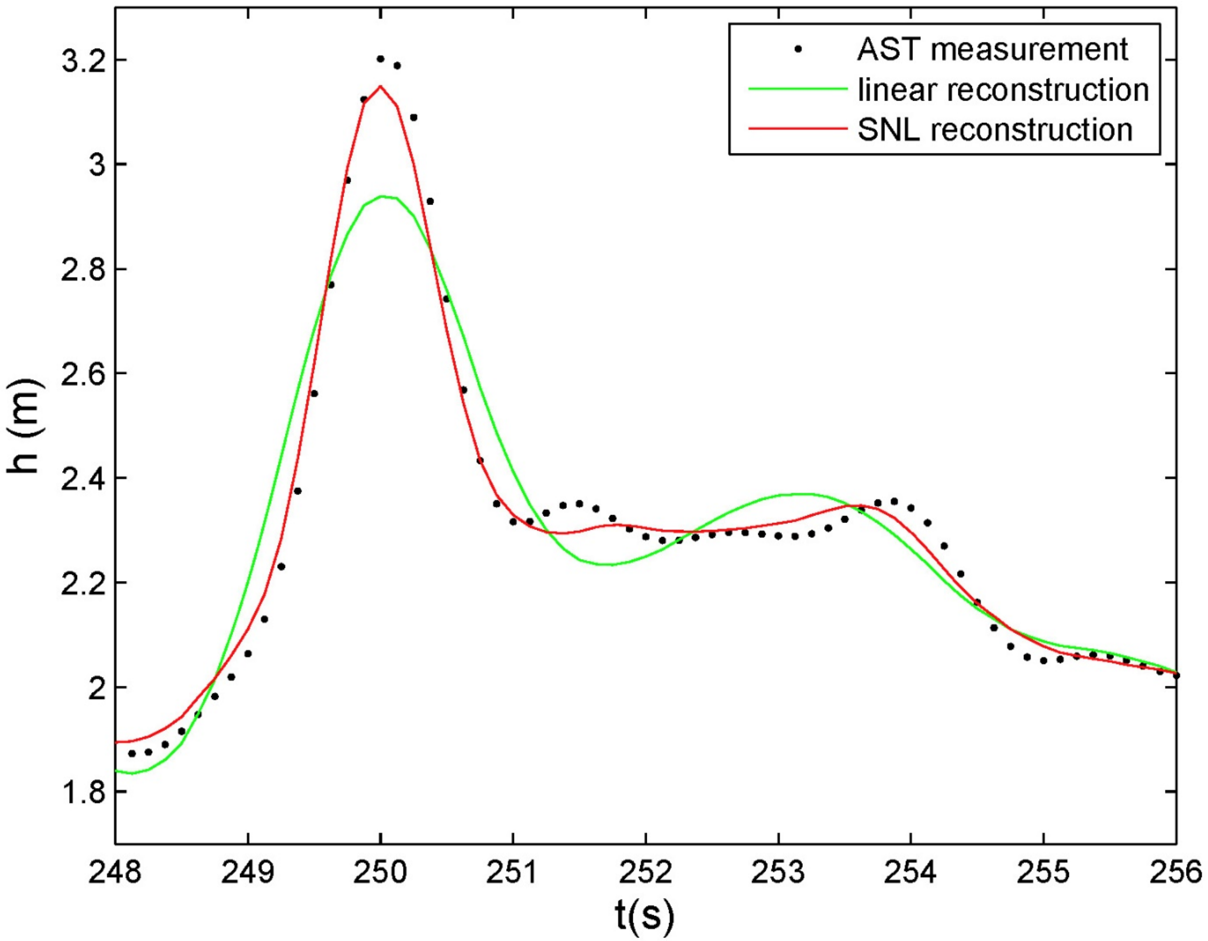
Field campaign

$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g}\partial_t(\zeta_{SL}\partial_t\zeta_{SL})$$

$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g}\partial_t^2\zeta_H$$

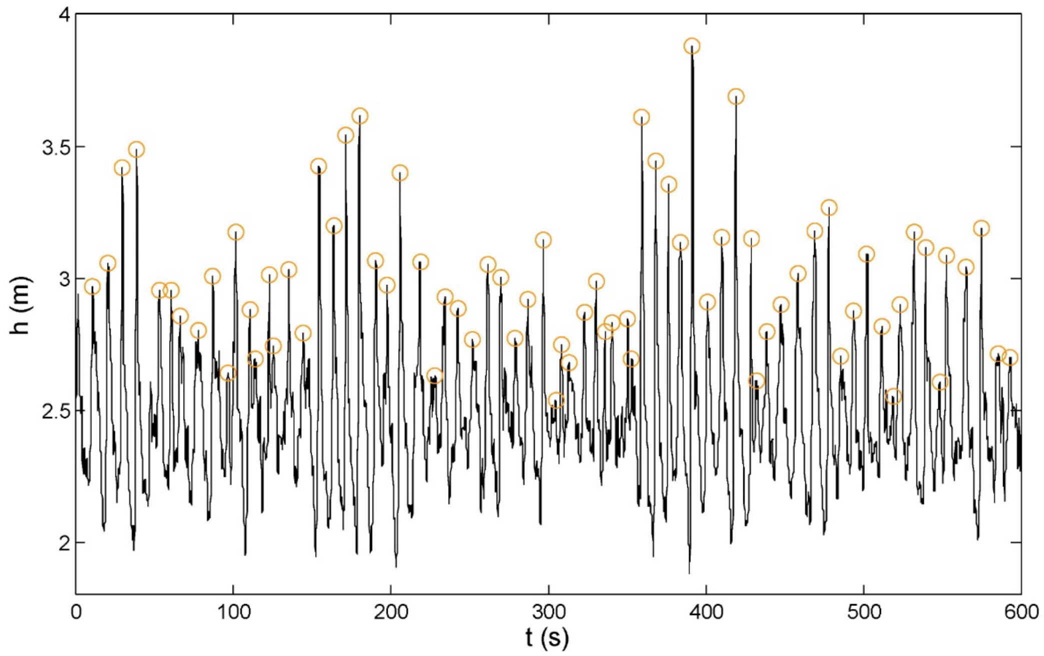


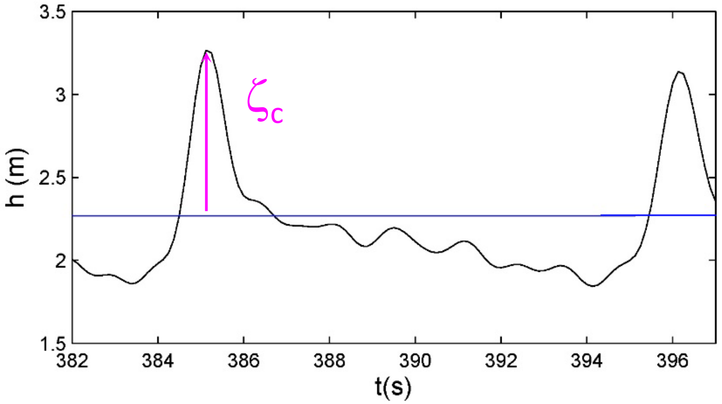
Field campaign

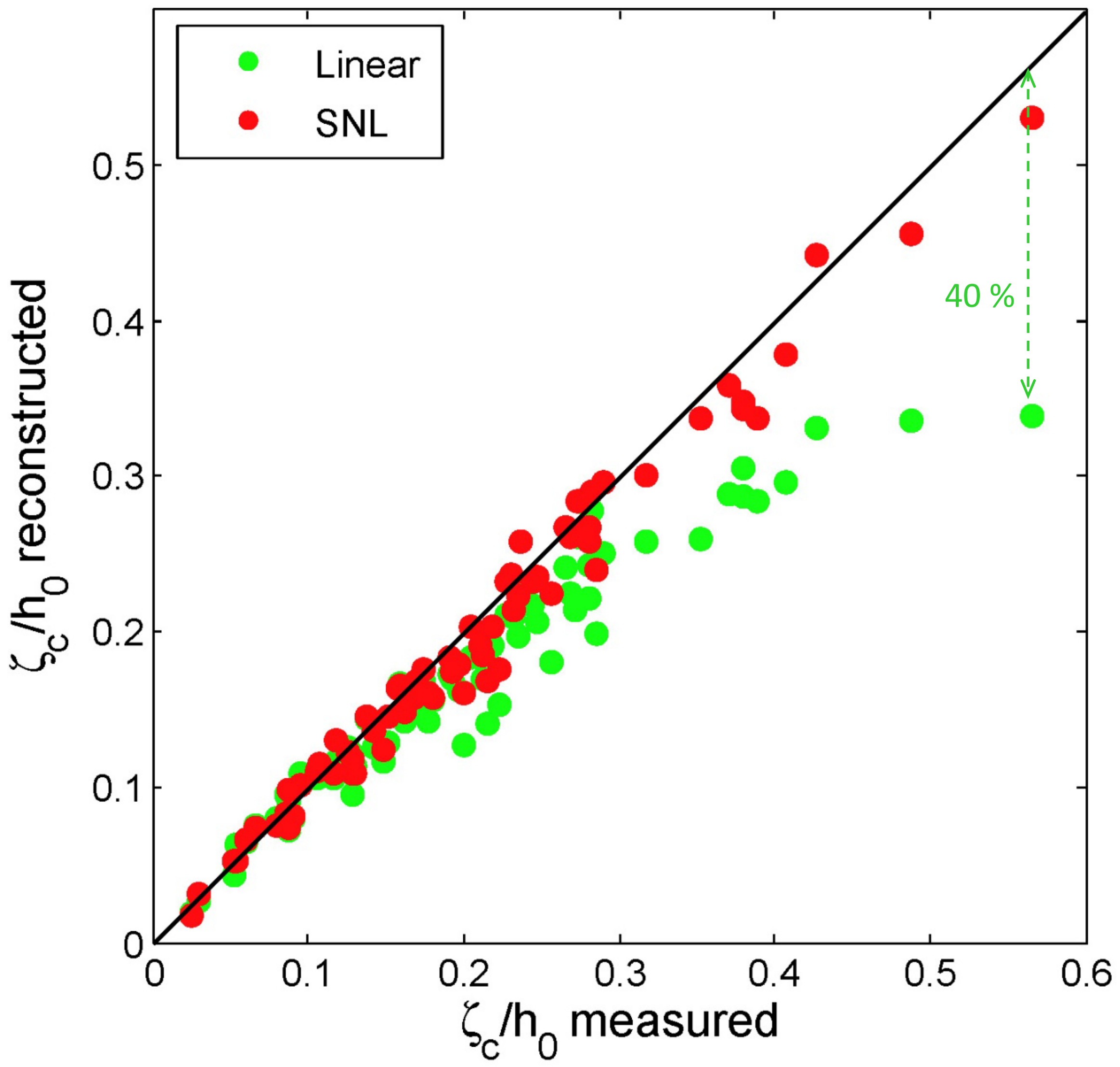


Field campaign

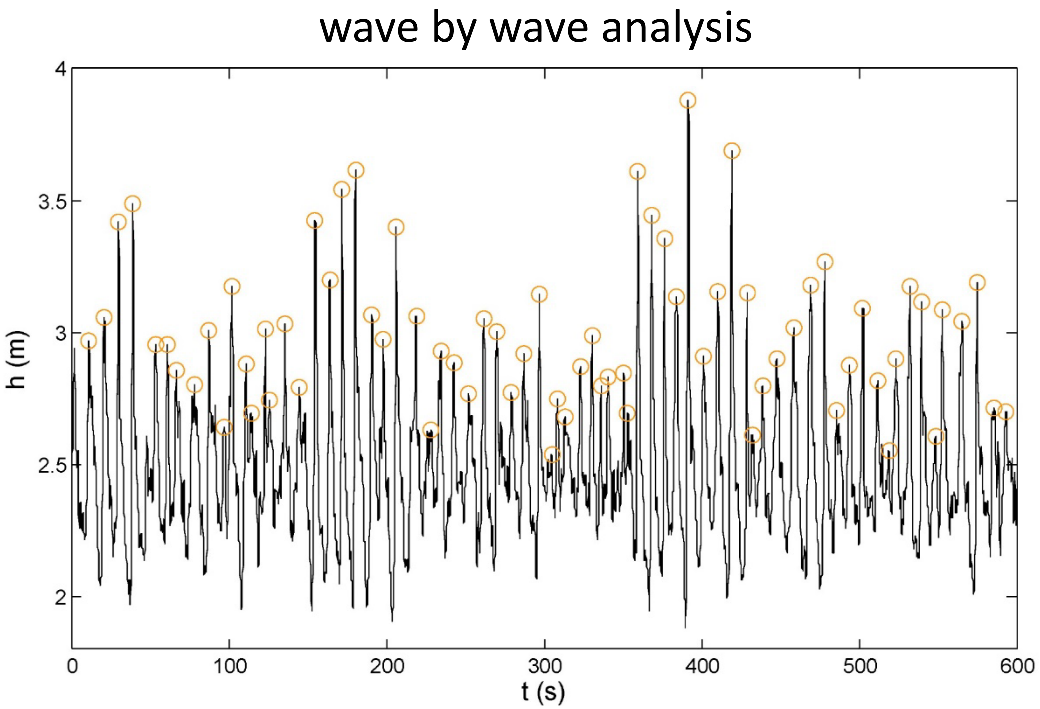
wave by wave analysis

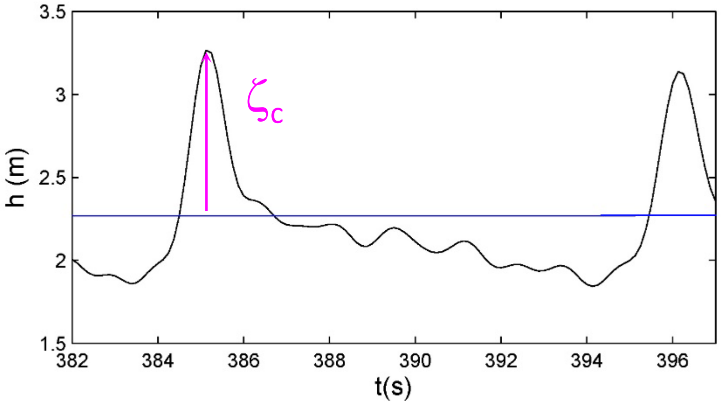


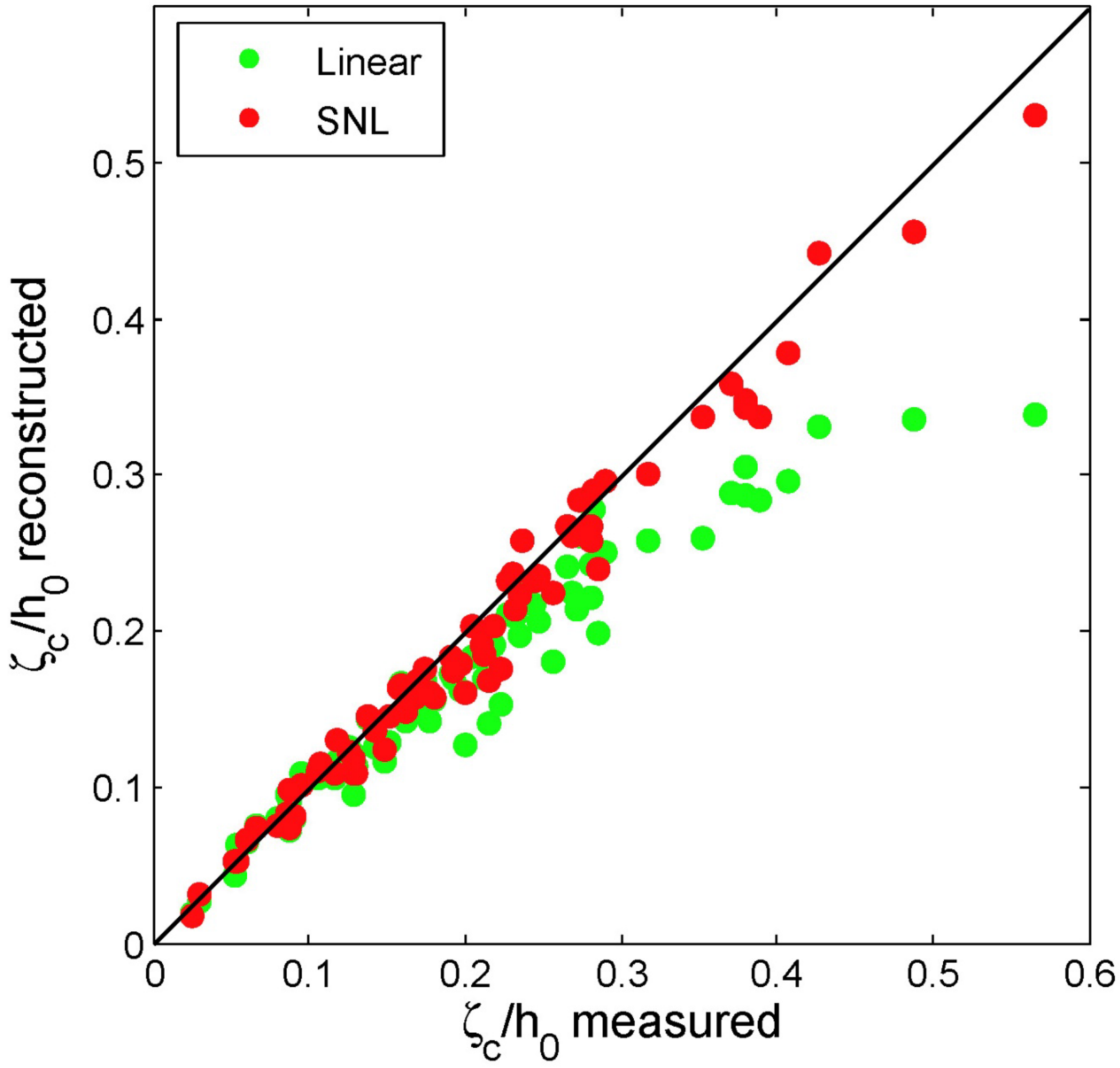




Field campaign



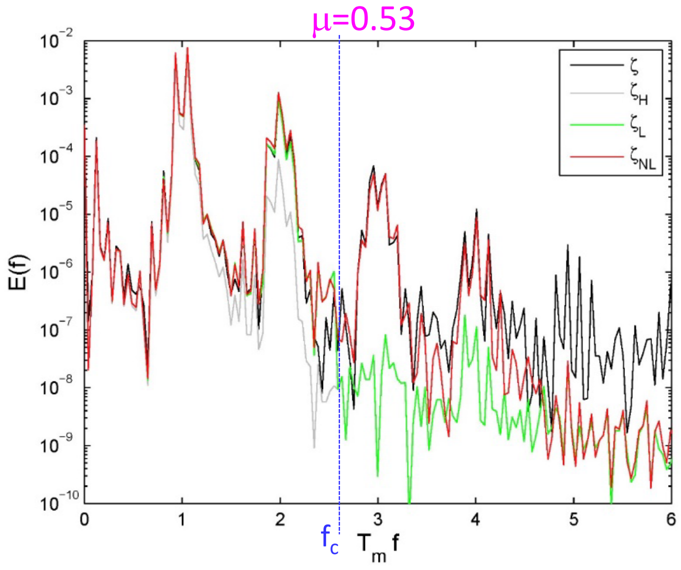




Two novel nonlinear methods for recovering the surface wave elevation from pressure measurements

❑ general fully dispersive method

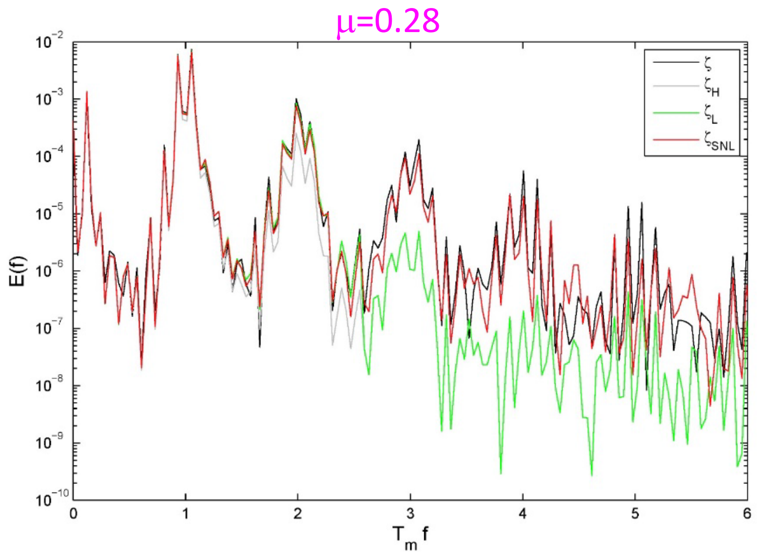
$$\zeta_{NL} = \zeta_L - \frac{1}{g}\partial_t(\zeta_L \partial_t \zeta_L)$$



❑ weakly dispersive method, $\mu \leq 0.3$

$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g}\partial_t(\zeta_{SL} \partial_t \zeta_{SL})$$

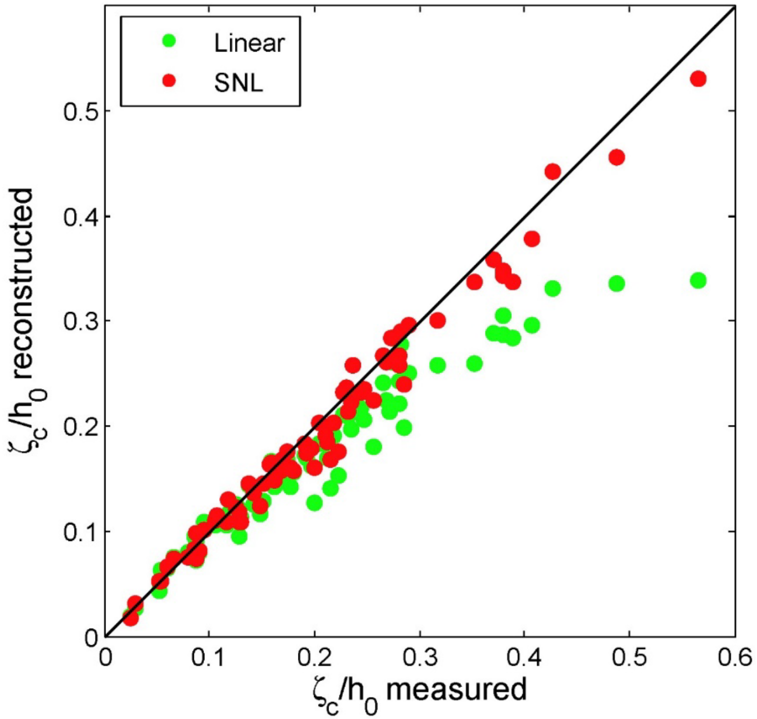
$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g}\partial_t^2 \zeta_H$$



Conclusion

Two novel nonlinear methods for recovering the surface wave elevation from pressure measurements

- provide much better results compared to the transfer function method commonly used in coastal applications
- are very simple and easy to use
- represent an economic alternative to direct wave elevation measurement methods (AST and Lidar)
- are a valuable tool for accurately characterizing extreme waves in shallow and intermediate water depths



Thank you for your attention